OPTION PRICING APPROACH TO INTERNATIONAL RESERVES

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Abstract

This paper brings forward the insurance aspect of holding reserves using the conceptual equivalence between insurance and financial options. By applying option pricing theory to approximate the cost of alternative precautionary arrangement that does not involve reserve accumulation and self-insurance, this paper explores when reserves are likely to become the primary means of precautionary arrangement, in particular in emerging markets.

Viewed in this light, the sharp rise in the amount of reserves held by many emerging markets can be traced to the rise in the "globalization hazard" that confronts emerging markets. A modest probability of globalization hazard (sudden stop) can induce emerging markets to self-insure fully by hoarding international reserves, rather than relying on non-reserve alternatives of taking precaution.

Keywords: precautionary need; reserve coverage; option and insurance; globalization hazard.

JEL codes: F31, F41.

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The real function of reserves is to give confidence...

Keynes on 24 September 1943[1]

1 Introduction

Following the financial crisis in 1997 and 1998, several Asian countries increased sharply their holdings of international reserves. International reserves held by Korea jumped from a little more than 5 percent of GDP in the early 1990s to 25-30 percent of GDP by 2002 and 2003 (Figure 1). Though not hit directly by the crisis, China's reserves increased from 10 percent to 25-30 percent of GDP over the same period.[2]

International reserves in the upwards of 20 percent of GDP are not unheard of. At an extreme end, Singapore has maintained international reserves that exceeded 50 percent of GDP since 1970, reaching 80-90 percent of GDP over the past ten years. As a more recent member of the club, Chile has maintained reserves in the range of 20-25 percent of GDP since the early 1980s.

In addition to the high level, one striking pattern—certainly for Korea—was the rapid rise in the level of reserves. Compared to the three decades before the Asian crisis, there was a virtual jump in the level of reserves in percent of GDP (Figure 2).[3] One possible cause is the rapid increase in the precautionary demand for international reserves. Indeed, several papers explore the extent to which precautionary motives can explain the high level of reserves held by many emerging-market countries following the recent financial crises. Aizenman and Lee (2007) show that the reserve accumulation was more closely correlated with variables that reflect precautionary motives than with variables that reflect mercantilist motives. Caballero and Panageas (2005) construct and calibrate a quantitative model of sudden stop that can justify the reserve accumulation that is observed in many emerging markets in recent years.

A related and no less important question is why a rise in precautionary need is accommodated primarily through the accumulation of reserves. Many precautionary needs in the economy are met

[1]The excerpt is from R. Skidelsky, *John Maynard Keynes: Fighting for Freedom, 1937-1946*, (Penguin Putnam: New York, 2000). This remark was made in the bilateral meeting between Britain and the U.S. that discussed the proposals for the Clearing Union (by Keynes) and the Stabilization Fund (by White).

[2]Since about 2003, reserves accumulation in China pulled ahead, which is discussed in Aizenman and Lee (2006).

[3]It may be associated with the desire to keep the exchange rate appreciation under control, but the Korean exchange rate did not become several times more rigid after the Asian crisis.

Figure 1: Korea and China: Reserves to GDP ratio

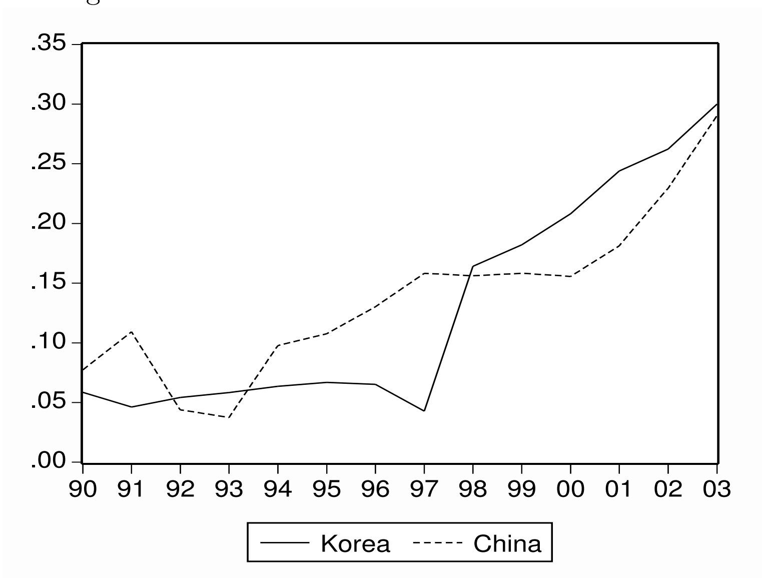
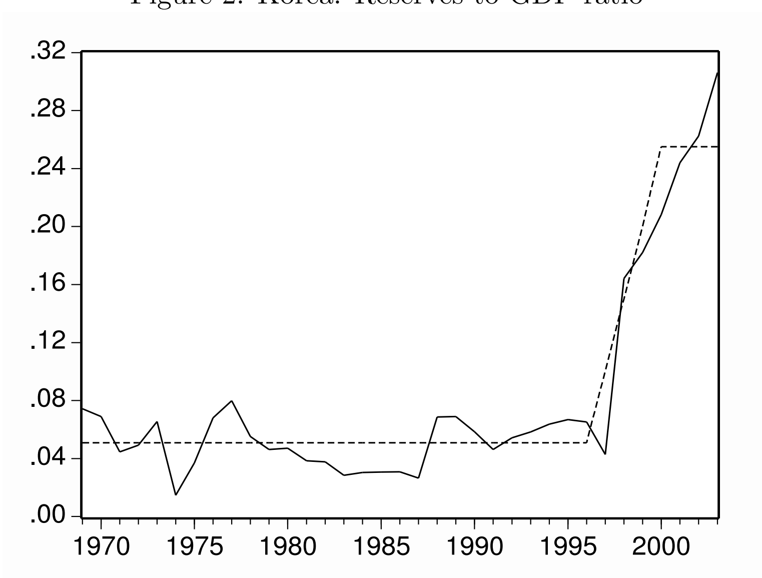


Figure 2: Korea: Reserves to GDP ratio



Dotted line denotes the averages over 1969-1996 and 2000-2003.

through explicit and implicit insurance arrangements, including (contingent) credit lines. And the financial market—a primary function of which is to facilitate risk sharing and insurance—continues to introduce innovative ways of sharing and trading risks across the market.

The decision to hold international reserves is an act of self-insurance, in that reserves are under the full discretion of the countries that hold them—reserves are an ultimate credit line, which can be tapped under any circumstance. Viewed this way, reserves stand at one end of the wide spectrum of credit lines subject to different contingencies, and question can be raised on what accounts for the apparent preference for a "corner solution" over and above alternative modes of precaution.

This question has received limited attention in the literature. With the exception of Caballero and Panageas (2005) and Lee (2004), existing models assume reserves to be the only means of precaution. For example, typical buffer-stock models of reserves solve for the optimal level of reserves under the assumption that reserves are the sole means of financing in times of adverse shocks (modeled as a diffusion process); so are most other papers on optimal level of reserves.

In contrast, this paper considers the possibility that countries can avail themselves of alternative means of taking precaution, and zooms in on the "reserve coverage ratio," defined as the optimal mix between reserves and alternative means of precaution. Disregarding this dimension could distort both positive and normative analysis of reserve holdings. If the increase in reserves is the combined outcome of a higher reserve coverage ratio and a higher precautionary need, ignoring one over the other would lead to biased estimates of the causes of reserve hoarding and misguided prescriptions on whether or how to adjust the level of reserves.

The reserve coverage ratio is determined at a level that balances the costs of reserves and alternative means of precaution, which are in turn approximated by the costs of their most important function. The cost of holding reserves is approximated by the interest rate spread on reserves, subject to several caveats that are acknowledged in the text. The generic cost of alternative means of precaution is approximated by the cost of a put option which provides an equivalent insurance. This follows Merton (1977 and 1998) who noted the equivalence between insurance and a put option in his analysis of the cost of providing deposit insurance.

With the cost of alternative means of precaution being approximated by the price of a put option written on a latent underlying asset, the stochastic process for the value of the underlying asset becomes the critical parameter for determining the reserve coverage ratio. A diffusion process, which was often used in the literature on reserves, has indeed been a good approximation when trade (current account) flows have been the main source of risks to external liquidity. As the integration of financial markets progresses, however, the risk of a sharp reversal in financial flows (aka sudden stop) has become more prominent, and this is probably better described as a jump risk. The existence of a jump risk raises sharply the reserve coverage ratio; the reserve coverage ratio rises from a value smaller than 0.5 to 1 for a sufficiently large jump risk, helping to understand both the jump in reserves of several Asian countries in recent years, and the generally high levels of reserves in many emerging markets that accompanied capital account liberalization. That is, the observed jump in reserves can be the combined outcome of the increase in the overall level of precautionary demand, and the rise in the ratio of precautionary demand that is accommodated by holding reserves (reserve coverage ratio).

The rest of the paper is organized as follows. Section 2 provides a short literature review on reserves to place this paper in the context, and discusses the notion of the reserve coverage ratio. Section 3 discusses the factors that influence the costs of self-insurance (reserves) and other non-reserve modes of precaution. Section 4 presents the illustrative calculation of the reserve coverage ratio, and is followed by a concluding discussion in Section 5.

2 International Reserves and Financial Options

2.1 Insurance Views of Reserves

Though not always couched in such terms, official reserves are ultimately held to guard against an undesired shortfall in international liquidity and to mitigate its adverse consequences. At the same time, holding reserves entails an opportunity cost, namely the interest rate cost. This fits the conceptual framework of insurance: guarding against a downside risk at the cost of insurance premium. The conceptual parallel has been noted implicitly in the traditional literature on reserves that has developed in parallel with the theory of the demand for money. To borrow terminology from money demand, reserves were viewed to be demanded largely for two motives, transactions motive and precautionary motive, the latter of which reflects insurance motive.

By initiating the analysis of reserve demand out of precautionary motive, Heller (1966) gave emphasis to the role of uncertainty. Since the ready availability of international liquidity would limit the extent of a downward adjustment that is needed in times of a deficit in the external balance, the monetary authorities would be inclined to hold international reserves out of precautionary motive. The optimal reserve holding would be mainly affected by the cost of adjustment in times of an external imbalance, the cost of maintaining a stock of reserves, and the probability of having to rely on international reserves.

Subsequent literature on reserves developed by elaborating on the nature of adjustment and uncertainty that were involved. In a prime example of the line of research that purported to improve the analysis of adjustment involved, Clark (1970) developed a general equilibrium model which illustrated the tradeoff between domestic adjustment and external financing (via reserves). The other line of research purported to analyze better the consequence of the degree of uncertainty involved; drawing on the theory of stochastic inventory control, Frenkel and Jovanovic (1981) developed a stochastic model in which depletion of reserves was assumed to impose a discrete (fixed) adjustment cost.

The insurance interpretation of reserves came out most clearly in the literature spawned by the currency crises of the 1990s. Contemplating on the possibility of currency crisis not warranted by fundamentals, Guidotti and Greenspan remarked on maintaining reserves that are sufficiently large as to exceed short-term external liabilities (Mulder (2000)). A similar war-chest motive of holding reserves was also advocated by Feldstein (1999) and Caballero (2003). In an analysis of politico-economic determinants of demand for reserves, Aizenman and Marion (2004) explicitly brought out the insurance value of reserves.

More recently, several papers have explored the extent to which precautionary motives can explain the high level of reserves held by many emerging-market countries. Aizenman and Lee (2006) show that the reserve accumulation is correlated with variables that reflect precautionary motives. Caballero and Panageas (2005) construct and calibrate a quantitative model of sudden stop and reserves hoarding. Garca and Soto (2004) and Jeanne and Ranciere (2006) calculated the optimal level of reserves, by calibrating models that build upon Ben-Bassat and Gottlieb (1992).

2.2 Alternative Means of Insurance

This paper explores a related but different question. What determines how strongly reserves are preferred as the means of precaution? One requirement for the optimal reserve-holding decision will be to strike the right balance in the cost tradeoff between cash reserves and non-cash means of precautionary arrangement. This condition will need to be satisfied regardless of the overall level of precautionary arrangement. The optimal mix of cash and non-cash means of precautionary arrangement is to the optimal level of precautionary arrangement, as cost minimization is to profit maximization, or as the optimal basket of consumption is to the optimal level of consumption: the former is a part or pre-condition of the latter.

Consider an economy that has a need for precautionary arrangement of size D_t. Holding cash reserves is the most conservative means of making such a precautionary arrangement. Other less conservative means would rely on the financial market in varying degrees, encompassing insurance arrangements of different sizes and terms. The examples range from financial hedging instruments, which are most direct and visible, to implicit hedging arrangements that include natural hedges and a favorable currency denomination for financial transactions in international financial markets. In between, the alternatives comprise many financial market arrangements (that facilitate risk sharing, including commercial credit lines and equity market itself).

The cost of non-cash means of precautionary arrangement can be viewed as the cost of obtaining an insurance of comparable size. Recognizing the put option element inherent in insurance arrangements, we use the price of put option to approximate the cost of non-cash means of precaution. When D_t is the desired level of precautionary arrangement and R_t is the level of international reserves, non-cash precautionary arrangement of size $D_t - R_t$ is needed. The cost of such non-cash arrangement is viewed as the price of a put option with exercise price $D_t - R_t$, written on a latent underlying asset of value V_t. Denoting the price of the hypothetical put option by $G(V_t, D_t - R_t)$ and the cost of carrying reserves by $c(R_t)$, the least-cost way for precautionary arrangement of level D_t is obtained by minimizing the overall (insurance) cost of precautionary arrangement:

$$IC(R_t; D_t) \equiv G(V_t, D_t - R_t) + c(R_t). \tag{1}$$

Using put option to calculate the cost of a precautionary arrangement is motivated by the seminal work of Merton (1977). He noted that the deposit insurance, viewed as a security, is isomorphic to a put option, and thus proposed calculating the cost of deposit insurance on the basis of the option pricing theory. The cost of various alternative means of precaution can thus be approximated by the price of the put option embedded in precautionary arrangements.

In comparing the two alternatives, the underlying asset is the anchor that links the alternative means of precaution and the inherent put option element. The identity and value of the underlying asset, however, are not fixed but vary with the insurance function that is fulfilled in different instances of precautionary arrangement. Most generally, we view the underlying asset as a latent variable that summarizes the external funds that are available to an economy, which thus can be supplanted by international reserves. While an economy will have a variety of precautionary needs, those that can be addressed by international reserves would be associated with external funds that are available to the economy.

The unobservability of the underlying asset makes it particularly difficult to determine its level, which is an important input into the option price. In this paper, the value of the underlying asset is assumed to be identical to the level of desired precautionary arrangement. If the current value of the underlying asset exceeds or falls short of the level of desired precautionary arrangement, the precautionary arrangement will begin to have the characteristic of an investment for profit or speculation, rather than a means of precaution.

3 Determinants of the Cost of Insurance Arrangement

Having postulated a particular level for the value of the underlying asset, we turn to risk characteristics of the underlying asset and cost elements of self-insurance. The risk associated with the latent underlying asset varies with the external environment for finance and trade.

3.1 BOP Interpretation and Underlying Asset

The option pricing framework for precautionary arrangement can be interpreted in terms of the familiar balance of payments (BOP) accounting. This helps to relate our framework to the traditional work on reserves and to motivate the discussion of risk characteristics of the underlying asset that follows.

Consider the following accounting identity for external balance. When R_t denotes reserves, IF_t denotes gross inflow and OF_t denotes gross outflows, we have:

$$\Delta R_t = IF_t - OF_t. \tag{2}$$

Assuming that the stock of reserves and the obligations for external payment are subject to less risk than the external revenue, we can reinterpret the gross inflow IF_t as the underlying asset V_t subject to risk, and the gross outflow OF_t as the debt or insurance need, denoted as D_t. In the world of the 1970s when the primary risk lied in trade flows, V_t could be viewed as the risky stream of export revenue and D_t could be viewed as the less risky stream of import expenditure. In today's world where the primary risk lies in financial flows, V_t can be viewed as gross inflows—comprising export revenue and financial inflows—and D_t could be gross outflows. This framework easily accommodates the roll-over risk of the short-term external debt that has been much discussed in crisis literature. The maturing liability obligations belong to D_t and the inflows from risky rollover belong to V_t.

A country can meet its external obligation D_t by combining reserves R_t and a precautionary arrangement of size $D_t - R_t$. The latter can be simply the value of the underlying asset V_t itself, or more efficiently, an arrangement that allows the country access to $D_t - R_t$. The cost of such an arrangement can be approximated as the cost of a put option written on V_t with exercise price of $D_t - R_t$, which brings us back to equation (1).

3.2 Globalization Hazard

Calvo (2005) observed that emerging markets exposed themselves to the vagary of international capital market. Many emerging market crises were characterized by a sudden stop, under which capital inflow dries up forcing a sharp current account adjustment on recipient countries. The sudden stop is not necessarily the outcome of neglect on the part of recipient country. To distinguish it from the moral hazard on the part of borrowing countries, Calvo suggested the term "globalization hazard" to capture the risk of a sudden stop. It is the hazard of participating in international capital markets.

We distinguish the globalization hazard from other external risks. Algebraically, globalization hazard is modeled as a rare and discrete event, the magnitude of which is larger than other external risks emanating from current account transactions. To borrow the exposition by Merton (1992), the distinction can be made mathematically concrete in the following manner.

Let time interval $[0, T]$ be divided into n intervals of length h ($T = nh$). The change in latent variable V_t is the sum of incremental change per unit interval:

$$V(T) - V(0) = \sum_{k=1}^{n} \left[V(k) - V(k-1) \right]. \tag{3}$$

Denote the incremental change by $\epsilon(k) \equiv V(k) - V(k-1)$, and assume that $\epsilon(k)$ can take values ϵ_j with probability p_j, for $j = 1, 2, \cdots m$.

Uncertainty that does not vanish too quickly can be modeled by assuming that $\sum_1^m p_j \epsilon_j^2$ is proportional to h. As time interval shrinks ($h \to 0$), the uncertainty declines at about the same rate as the time interval, lending itself to a meaningful continuous time representation. For the variance per time h to be proportional to the time length h, the larger is the magnitude of change $|\epsilon_j|$, the smaller is its probability p_j. Within that class, however, two qualitatively different cases can emerge, with the difference lying in the magnitude of possible change.

Merton shows that when the magnitude of change $|\epsilon_j|$ is smaller than a certain threshold, $\epsilon(k)$ lends itself to a representation by the familiar diffusion process. There is zero probability of no change over a short time interval, while each change is small enough to produce a continuous sample path. This provides a reasonable description of risks faced by an economy exposed to shocks in the trade account.

When the magnitude of change exceeds a certain threshold, $\epsilon(k)$ can be represented as a jump process. There is substantial probability of no change over a very short interval, but the change, when it arises, becomes so large as to bring about a discontinuous sample path. The change is rare but large. Globalization hazard is modeled as a rare event, which corresponds to a rare and discrete jump in the value of the underlying asset.

Without the globalization hazard, the underlying asset value is assumed to follow a standard diffusion process:

$$\frac{dV_t}{V_t} = \mu dt + \sigma d\omega(t), \tag{4}$$

with $\omega(t)$ denoting a standard Brownian motion. The increment of the diffusion process has instantaneous mean μ and variance σ^2. With the globalization hazard, the value of the underlying asset is assumed to follow a jump-diffusion process.

The jump component is captured by a Poisson process with arrival probability of λ. To elaborate, over time interval $(t, t+h)$, an event (sudden stop) occurs with probability λh. Under the event, the value of the underlying asset makes a discrete jump of the magnitude determined by another random drawing, which in turn is summarized by a random variable Y, $V(t+h) = V(t)Y$. Superimposing this jump risk on the diffusion process in equation (4), Merton (1992) showed that $V(t)$ can be written as the following stochastic differential equation, where dq_t denotes the Poisson process we just described and $k \equiv E(Y-1)$:

$$\frac{dV_t}{V_t} = (\mu - \lambda k)\, dt + \sigma d\omega_t + dq_t \tag{5}$$

3.3 Cost of Holding Reserves

The economic cost of self-insurance by holding international reserves comprises two components: the opportunity cost and the convenience yields of holding reserves. The opportunity cost of reserves has often been measured by the interest rate spread (financial carry cost), and we will stay close to that practice, regarding it as a good approximation to financial opportunity cost.

However, a discussion is in order as regards finer points of the ultimate cost of holding reserves. The economic—maximum—opportunity cost depends on the best alternative use of resources tied in reserves, and can substantially exceed the financial opportunity cost measured by spreads (see Hauner (2005) for a calculation of the economic opportunity cost and Caballero (2003) for an earlier discussion of a related issue). Our analysis, however, focuses on the financial aspect of international reserves, and squares with the measure of financial opportunity cost.

The convenience yields of holding reserves reduce the cost of self-insurance below the usual opportunity cost of reserves. In contrast to an insurance arrangement stipulated for a particular contingent event, reserves can be used for a variety of contingencies. This flexibility benefit of holding reserves corresponds to the concept of convenience yields, and lowers the economic cost of self-insurance by the same amount.[4]

The economic cost of self-insurance by holding international reserves is thus obtained by subtracting the convenience yields from the opportunity cost of carrying reserves. If the opportunity cost is higher than the financial carry cost, the convenience yields would reduce the gap between the

[4] See Krishnamurty and Vissing-Jorgenson (2007) for an effort to estimate the convenience yield for the U.S. government securities, which are indeed a dominant instrument for reserve portfolio.

economic cost of self-insurance and the financial carry cost. While acknowledging these refinements and qualifications, for the remainder of this paper, we approximate the cost of self-insurance by the financial carry cost of international reserves.

4 Reserve Coverage Ratio

To explore the quantitative implication of the framework developed so far, we turn to the Black-Scholes formula (Black and Scholes (1973) and Merton (1992)). From the viewpoint of quantitative accuracy, the parameterization by a European option is an approximation. However, the European option price captures the lion's share of the market value of insurance, and offers a flexible apparatus for the quantitative framework proposed in this paper.[5] The availability of a closed-form solution also makes it easy to illustrate the role of globalization hazard (jump risk). Moreover, under the Black-Scholes formula, the insurance value and the optimal reserve coverage ratio depend on the ratio of D (or R) to V, independent of the level of V. (The time subscript t will be suppressed in the rest of the paper for simplicity of notation.) Combined with our assumption that $V = D$ that was discussed earlier, the risk characteristics play the most important role in determining the degree of self-insurance (reserve coverage ratio).

4.1 Hazard Free

Let $g(V, E, \tau)$ denote the price of a put option with exercise price E when the value of the underlying asset follows a log-normal process of equation (4). When self-insurance accounts for R out of the overall insurance need D, the cost of alternative modes of precaution amounting to level $D - R$ is the price of a put option with exercise price $D - R$:

$$g(V, D - R, \tau) = (D - R)e^{-r\tau}N(-d_2^R) - VN(-d_1^R) \tag{6}$$

$$\text{where} \quad d_1^R = \frac{\log(\frac{V}{D-R}) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{and} \quad d_2^R = d_1^R - \sigma\sqrt{\tau}.$$

[5] Several financial instruments offer richer insurance possibilities than a European option. An American option allows an early exercise prior to the expiration date, and credit derivatives offer insurance against credit events that are covered by the contract. These instruments with greater flexibility, however, would be more costly, and the European option price can thus be viewed as providing the lower-bound approximation of the actual cost of a variety of alternative precautionary arrangements.

The total cost of precautionary arrangement is:

$$IC(R) \equiv g(D - R, V, \tau) + c(R) \tag{7}$$

The first-order condition for the optimal reserve coverage (self-insurance) is then:

$$MIC(R) = -e^{-r\tau} N(-d_2^R) + c'(R) = 0. \tag{8}$$

From equations (6) and (8), it is clear that the optimal reserve coverage ratio depends on the ratio of D (or R) to V, independent of the level of V.

Proposition 1 *The optimal reserve coverage ratio is determined as an internal solution when* $0 < c'(R) < 1$ *for* $R \in [0, D]$.

The proof contained in the appendix relies on the basic properties of the (European) put option. The marginal cost in equation (8) is an increasing function of R, reflecting the convexity of the European option price. Hence, we have only to show that $MC(R = 0) < 0 < MC(R = D)$. The first inequality follows directly from the assumption of the proposition, and the second inequality can be proved by combining the assumption with the fact that $\lim_{R \to D}(-d_2^R) = 0$.

Proposition 1 implies that full self-insurance is not an optimal choice. At a low level of self-insurance, the marginal decline in the market price (cost) of insurance is large enough to offset the increase in the carry cost; a higher self-insurance (reserve holding) reduces the total cost of precautionary arrangement. As the share of self-insurance rises, however, the marginal decline in the market price (cost) of insurance shrinks, and to be dominated by the increase in the carry cost beyond a threshold—raising self-insurance beyond that point increases the overall cost of precautionary arrangement, and the optimal reserve coverage ratio is determined at that point.

4.2 Under Globalization Hazard

In addition to the basic specification of jump-diffusion risk in equation (5), we make the simplest assumption that the value of the underlying asset falls to zero when the globalization hazard (or sudden stop) materializes. Under this assumption, the price of call option takes a highly tractable form (see the appendix for the general formula). When the price of call option on an asset without jump is denoted by $f(V, E, \tau, r, \sigma)$, the price of call option on an asset with jump probability λ is:

$$F(V, E, \tau, r, \sigma, \lambda) = f(V, E, \tau, r + \lambda, \sigma) \tag{9}$$

The jump probability (globalization hazard) has the same effect on the call-option price as a rise in the (risk-free) interest rate.

The price of put option on the asset with exercise price E is derived by substituting equation (9) into put-call parity:

$$G(V, E, \tau, r, \sigma, \lambda) = F(V, E, \tau, r, \sigma, \lambda) - V + Ee^{-r\tau}. \tag{10}$$

The overall cost of precautionary arrangement is:

$$IC(R; D) = G(V, D - R, \tau, r, \sigma, \lambda) + c(R). \tag{11}$$

The hazard rate changes the marginal insurance cost through its effect on the option price.

$$\frac{\partial G}{\partial R} = e^{-(r+\lambda)\tau} N\left(d_2^R + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) - e^{-r\tau} \tag{12}$$

The effect of the hazard rate on the marginal insurance cost is not monotonic. As the hazard rate increases, the probability for the insurance disbursement to occur (option exercised) rises (the term $N(..+\lambda..)$), which tends to raise the marginal insurance cost through non-reserve alternatives. As the result, it tends to increase the appeal of self-insurance from the viewpoint of minimizing the total cost of precautionary arrangement. To write out the marginal cost of additional reserve coverage,

$$MIC(R) = \frac{\partial G}{\partial R} + c' = e^{-(r+\lambda)\tau} N\left(d_2^R + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) - e^{-r\tau} + c'. \tag{13}$$

Now even at a relatively high level of self-insurance, the marginal decline in the market cost of insurance (which is the negative of the marginal insurance cost, or, mathematically, the increase in the put option price) can be large enough to offset the increase in the carry cost. When the globalization hazard is sufficiently large, the marginal insurance cost ($|\frac{\partial G}{\partial R}|$) unambiguously exceeds the cost of holding reserves, thereby pushing the optimal reserve coverage ratio to equal 1.

Proposition 2 *The optimal reserve coverage ratio equals 1 when the globalization hazard is sufficiently larger than the carry cost of holding reserves.*

In the presence of globalization hazard, it can be optimal for the insurance to be funded fully by holding international reserves. Allowing for this possibility, the first-order condition for the reserve coverage ratio in equation (1) is as follows. (See the appendix for the full algebraic proof.)

$$\left(\frac{\partial G}{\partial R} + c'\right)(D - R^*) = 0 \tag{14}$$

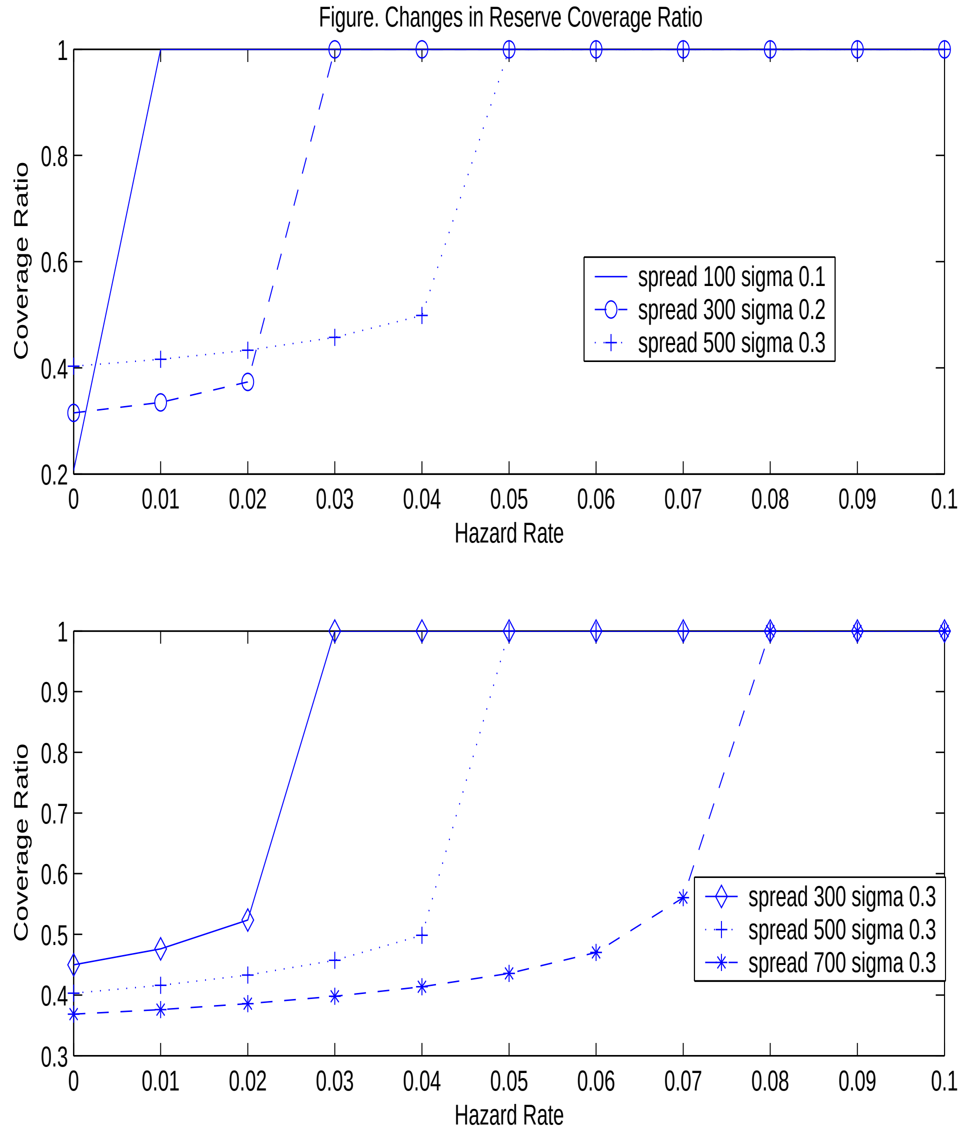
and

$$\frac{\partial G}{\partial R} + c' \leq 0 \quad R^* \leq D. \tag{15}$$

This condition reduces to equation (8), the first-order condition without globalization hazard when $\lambda = 0$. In the presence of a positive globalization hazard, $\frac{\partial G}{\partial R}$ is negative for all values of d_2^R. Even when d_2^R increases to a very large value as the reserve coverage approaches 1 (R approaches D), the marginal decline in the price of a put option ($|\frac{\partial G}{\partial R}|$) remains strictly above zero. Even at a very low exercise price, the possibility of a discrete jump in the value of the underlying asset keeps the marginal increase in the put-option price strictly above zero, rather than having it converge to zero in the absence of the possibility of a discrete jump.

The overall marginal cost of increasing reserve coverage in equation (13) can remain negative for all levels of reserve coverage, if the globalization hazard is sufficiently large relative to the marginal carry cost of holding international reserves. The hazard rate (jump risk) creates a non-zero wedge in the marginal decline in the put option price, as can be seen in equation (12) or (13). When the hazard rate is positive, even as the reserve coverage ratio is close to 1, the marginal cost effect of changing reserve coverage is strictly different from zero. It is intuitive enough that the complete reserve coverage is more likely when the hazard rate is large relative to the marginal carry cost. In particular, the sufficient condition of Proposition 2 can be interpreted as implying that the hazard rate exceed the marginal carry cost (or spread). Figure 3 illustrates the rapid jump in the reserve coverage as the hazard rate crosses the threshold that is equal to the interest rate spread.

Figure 3: Coverage Ratio, Spread, and Hazard Rate



5 Discussion and Conclusion

We have noted that the reserve coverage ratio can rise rapidly with the globalization hazard, when the hazard rate is large relative to the spread. Figure 4 illustrates it for three different hazard rates (0, 0.02, and 0.05). A possible parallel between this channel and the experience of Korea can be gleaned from Figure 5 that plots the combination of reserves and the spread over the 1998-2004 period. The figure suggests that a sharp increase in the reserve coverage could have followed, as the rise in the perceived hazard risk met with the propitious decline in the spread in the 2000s. Combined with the likely increase in the level of demand for precautionary arrangements, the rise in the optimal coverage ratio could have contributed to the rapid rise in the level of reserves held by emerging markets.

Placed in the broader literature of reserve demand, the reserve coverage ratio provides another channel that helps to determine the level of reserves that are held out of precautionary motive. The literature has mostly examined the desired level of reserves, treating reserves as about the only means of taking precautions against external shocks. However, considering that the self-insurance provided by reserves does not have to be the only or the most efficient means of taking precaution, we explored the effect of globalization hazard on the reserve coverage ratio.

Faced with the hazard of globalization, the full self-insurance can be an optimal financial decision, independent of the desired level of overall precautionary arrangements that is consistent with the globalization hazard. Thus, a rise in globalization hazard can increase reserve hoarding in two stages, first by increasing the desired overall level of precautionary arrangement, and second by increasing the reserve coverage ratio—the share of precautionary arrangement that is covered by hoarding international reserves.

This line of reasoning implies that, in the positive or normative research on reserves, an increase in precautionary reserve hoarding is a mix of higher level of desired precautionary arrangement and higher reserve coverage ratio for given levels of desired precautionary arrangement. Whereas combining the two could be a coherent way of measuring higher reserve demand, it leaves open the possibility of over-estimating the benefit and use of reserve accumulation in circumstances when the reserve coverage ratio changes.

The reserve coverage ratio was discussed only in terms of the nature of risk that faces the economy. Eventually, however, the reserve coverage ratio would be affected by the degree of financial

Figure 4: Theoretical Coverage Ratio and Spread

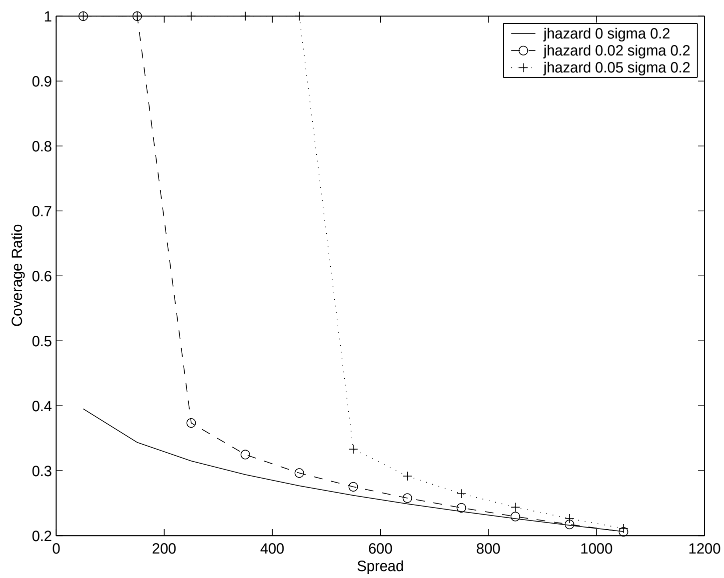
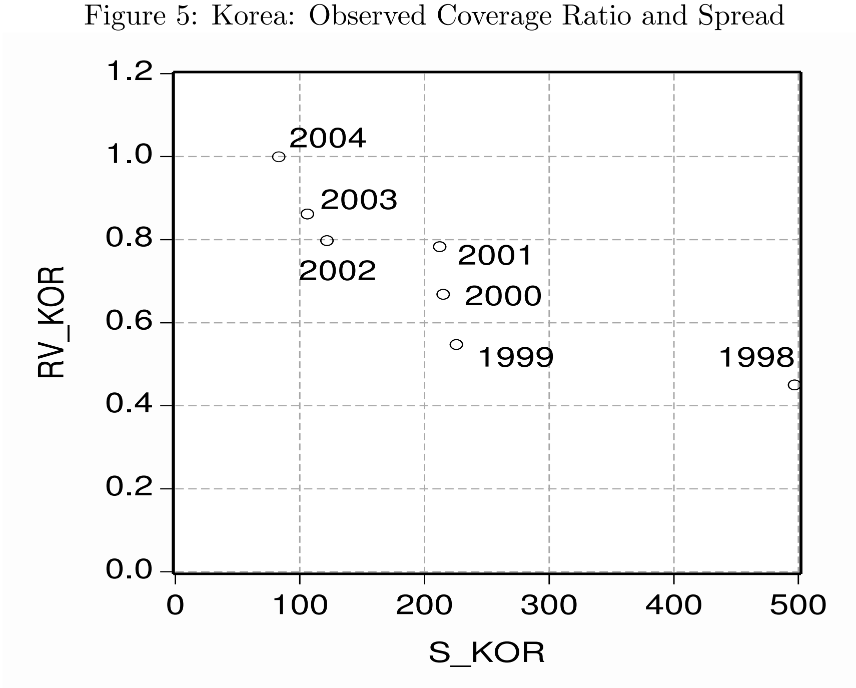


Figure 5: Korea: Observed Coverage Ratio and Spread



market development. Expansion in financial-market-provided means of risk sharing would lower the reserve coverage ratio. Even for the macroeconomic hazards that reserves are intended to cover, the market-mediated insurance can at some point be provided by the likes of macroeconomic insurance advocated by Shiller (2003). The development of these instruments would expand the set of market-based alternatives to self-insurance, and lessen the reliance on reserves. This aspect is particularly important in the normative analysis that aims at providing a prescriptive characterization of the optimal level of reserves, both for a quantitative analysis of optimal reserve levels and for an institutional analysis of the need for reserve holdings.

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A Option Prices with and without Jump

When the value of the underlying asset follows a log-normal process

$$\frac{dV_t}{V_t} = \mu dt + \sigma d\omega(t),$$

with $\omega(t)$ denoting a standard Brownian motion, the price of a put option with exercise price E is:

$$g(V, E, \tau) = Ee^{-r\tau} N(-d_2) - VN(-d_1) \tag{16}$$

$$\text{where} \quad d_1 = \frac{\log(V/E) + (r + \frac{\sigma^2}{2})\tau}{\sigma\sqrt{\tau}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{\tau},$$

where Ndenotes the standard normal distribution function.

The value of the underlying asset follows a jump-diffusion process. The diffusion component is assumed to follow a standard geometric Brownian motion with its increment having instantaneous mean μ and variance σ^2. The jump component captured by a Poisson process with arrival probability of λ. To elaborate, over time interval $(t, t + h)$, an event (sudden stop) occurs with probability λh. Under the event, the value of the underlying asset makes a discrete jump of the magnitude determined by another random drawing. If the random drawing is summarized by a random variable Y, $V(t + h) = V(t)Y$. When dq_t describes this Poisson process and $k \equiv E(Y - 1)$, Merton (1992) shows that $V(t)$ can be written as the following stochastic differential equation, where $d\omega_t$ denotes a Wiener process and dq_t denotes the Poisson process we just described.

$$\frac{dV_t}{V_t} = (\mu - \lambda k)\, dt + \sigma d\omega_t + dq_t \tag{17}$$

In our case, we will make the additional assumption that capital inflow dries up fully, when a sudden stop hits. Under that assumption, $Y \equiv 0$ and $k = -1$.

$$\frac{dV_t}{V_t} = (\mu + \lambda)dt + \sigma d\omega_t \text{ with probability } 1 - \lambda \tag{18}$$

$$= (\mu + \lambda)dt + \sigma d\omega_t - 1 \text{ with probability } \lambda. \tag{19}$$

Under this assumption, the price of put option on the asset with exercise price E is derived as follows:

$$
\begin{aligned}
G &= F(V, E, \tau, r, \sigma, \lambda) - V + Ee^{-r\tau} && (20)\\
&= VN\left(d_1 + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) - Ee^{-(r+\lambda)\tau}N\left(d_2 + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) - V + Ee^{-r\tau}. && (21)
\end{aligned}
$$

Differentiating the put-option price with respect to the exercise price D,

$$
\begin{aligned}
\frac{\partial G}{\partial E} &= \frac{\partial F}{\partial E} + e^{-r\tau} && (22)\\
&= \frac{\partial f(\cdots, r+\lambda)}{\partial E} + e^{-r\tau} && (23)\\
&= -e^{-(r+\lambda)\tau}N\left(d_2 + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) + e^{-r\tau} && (24)
\end{aligned}
$$

B Proofs of Propositions

Time subscript is suppressed, as in the corresponding text discussions.

B.1 Proposition 1

When self-insurance accounts for R out of the overall insurance need D, substituting $D - R$ into E in equation (16) leads to equation (7), the market [cost of insurance]. To repeat the condition for optimal reserve coverage using equation (8),

$$MC(R) = -e^{-r\tau}N(-d_2^R) + c'(R) = 0. \quad (25)$$

This forms the basis for Proposition 1.

- The optimal reserve coverage ratio depends on the ratio of D (or R) to V, independent of the level of V. Equation (25) shows that the optimal coverage ratio is determined by the expression within the bracket, which depends only on the ratio of $\widetilde{E}$ to V where $\widetilde{E} = D - R$.

- The marginal cost in equation (25) is an increasing function of R, as a higher value of R would lower $-d_2^R$. This property reflects the convexity of the European option price and algebraically, $\frac{\partial MC}{\partial R} = e^{-r\tau}N'(-d_2^R)\frac{-1}{\sigma\sqrt{\tau}} < 0$. ¶

- The optimal reserve coverage ratio is determined as an internal solution when $0 < c'(R) < 1$. Since the marginal cost function increases in R, we have only to show that $MC(R = 0) < 0 < MC(R = D)$. It is easy to see that $MC(R = 0) < 0$ under $c'(L) < 1$, which is likely to hold true in most conceivable cases. To see $MC(R = D) > 0$ when $c' > 0$, note that $\lim_{R\to D} -d_2^R = 0$. Hence, $\lim_{R\to D} MC(D - R) = -e^{-r\tau}N(-\infty) + c'(R) > 0$ when $c'(R) > 0$.

B.2 Proposition 2

$$
\begin{aligned}
\frac{\partial G}{\partial R} + c' &= e^{-r\tau}\left[e^{-\lambda\tau}N\left(d_2^R + \frac{\lambda\tau}{\sigma\sqrt{\tau}}\right) - 1\right] + c' \\
&\ll e^{-r\tau}\left[e^{-\lambda\tau} - 1\right] + c'. \qquad (26)
\end{aligned}
$$

Equation (26) is negative at $R = D$ when

$$
1 - e^{-\lambda\tau} \geq c'(D)e^{r\tau}. \qquad (27)
$$

To rewrite this condition in terms of λ,

$$
\lambda\tau \geq -\log\left[1 - c'(D)e^{r\tau}\right]. \qquad (28)
$$

When the [carry] cost of maintaining reserves amounts to

$$
\begin{aligned}
c(R) &= \left(e^{r^l\tau} - e^{r\tau}\right)e^{-r\tau}R \qquad (29) \\
&= \left(e^{(r^l - r)\tau} - 1\right)R = \left(e^{s\tau} - 1\right)R, \qquad (30)
\end{aligned}
$$

condition (28) becomes

$$
\lambda\tau \geq -\log\left[1 + e^{r\tau} - e^{(s+r)\tau}\right]. \qquad (31)
$$

Using two approximation formula, $e^x = 1 + x$ and $\log(1 + x) = x$, equation (31) becomes $\lambda \geq s$.