A Macroeconomic Perspective on Reserve Accumulation

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1. Introduction

In the aftermath of the Asian financial crises, there has been a massive accumulation of international reserves by emerging markets, particularly in East Asia. Economists have suggested three ways to explain this phenomenon. The first, drawing on a buffer stock framework, emphasizes the role of international reserves in providing self-insurance.[1] For emerging markets, reserves reduce the cost of a financial crisis and may also reduce the chance of crisis. Of course, these benefits must be weighed against the opportunity cost of holding reserves.

A second approach promotes the idea that a new Bretton Woods has emerged with a fixed exchange-rate periphery in Asia and the U.S. as center country. The development strategy of the periphery is one of export-led growth supported by undervalued exchange rates, capital controls and the accumulation of reserve asset claims on the center country.[2]

A third approach argues that Asian countries pursue soft dollar pegs because so much of their trade is invoiced in dollars and their international lending is in dollars. U.S. dissaving and current-account deficits force high-saving Asian countries to intervene in the foreign-exchange market to maintain their pegs, racking up current-account surpluses and reserves.[3]

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[1] Examples of models where reserves provide insurance are Van Wijnbergen (1990), Ben-Bassat and Gottlieb (1992), Aizenman and Marion (2003, 2004), Jeanne and Ranicere (2006), and Jeanne (2007).

[2] See Dooley, Folkerts-Landau and Garber (2003, 2007) and Dooley and Garber (2005). Although an export-led growth strategy need not require a current-account surplus, it is argued that reserves accumulated via current-account surpluses provide the needed collateral for efficient international intermediation of Asian domestic saving.

[3] See McKinnon and Schnabl (2004a, 2004b, 2006). This explanation extends to Japan.

To the best of our knowledge, only the first approach has been modeled explicitly. The standard buffer stock model captures the essence of the self-insurance motive.[4] That model says the reserve authority optimally chooses an initial upper bound for reserves to minimize total expected costs. Yet reduced-form regressions of the model as well as simulations suggest that reserve holdings in Asia far exceed the amount required to meet insurance needs. Also problematic, the model adopts the assumption that the reserve authority targets directly the level of reserves. In many cases, however, the reserve level is a by-product of a policy targeting other macroeconomic variables.

The main objective of this paper is to develop a model that nests the insurance approach with the export-led approach in order to demonstrate that the combination of the two can potentially explain the recent accumulation of reserves in Asia. In our model, the policymaker targets output and inflation, but may choose to target reserves as well. If output is below its natural rate, the policymaker adopts a weak currency to stimulate output and reserve accumulation is a by-product of the exchange-rate policy. This, in nutshell, is the export-led explanation of reserve accumulation.

At the same time, extreme reserve levels are costly. Low levels can yield costly crises and increase the probability of future crises, as emphasized by the insurance story. Low reserves may also constrain the policymaker to accept a weak currency to attract additional reserves, generating additional costs because the economy cannot achieve its first-best output and inflation targets. High reserve levels are also undesirable. They generate political pressures from trading partners, are difficult to sterilize fully, and increase the opportunity cost of holding reserves.

[4] For a review of the buffer stock model and some recent extensions, see Flood and Marion (2002) and Bar-Ilan, Marion and Perry (2007).

We consider two solution concepts. The first is discretion, where the policymaker responds to contemporaneous real and nominal shocks by adjusting the nominal exchange rate period-by-period. The second solution is a rule with escape clause, where the policymaker fixes the exchange rate but reneges when reserves get too low.

In previous models, reserves are either a by-product of a fixed exchange rate or chosen optimally without any concern for the exchange rate.[5] By introducing both exchange-rate policy and reserve targeting into an intertemporal macro model, we make explicit the policymaker's concern with both the exchange rate and reserves. We also make explicit the two-way linkages between the exchange rate and reserves. The exchange rate set by the policymaker can be sustained only if there is sufficient reserve backing. At the same time, reserve changes are determined, in part, by the exchange rate.

For both discretion and the rule with escape clause, the model gives solutions for the nominal and real exchange rate, output, inflation, and reserves. It also reveals the welfare loss. One can analyze these solutions in light of the Asian experience.

An interesting special case is that of China, which for much of the last fifteen years faced an excess supply of labor and output below potential. Under a rule, the policymaker finds it optimal to adopt a weak-currency peg and accumulate significant reserves. Under discretion, the policymaker also chooses a weak currency initially, possibly even a weaker currency than the peg, and accumulates significant reserves. But the welfare loss is smaller under discretion, at least when the output gap is large.

[5] For example, the original buffer stock model has the policymaker choose the optimal reserve stockpile without any reference to exchange-rate policy. Reduced-form regressions of the buffer stock model sometimes include nominal exchange-rate volatility as a regressor to test whether greater exchange-rate flexibility is negatively correlated with reserve holdings.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 describes the model solution for the case of discretion. Section 4 presents the solution for the rule with escape clause. Section 5 highlights aspects of the two cases. Section 6 analyzes some extensions of the basic model. Section 7 concludes.

2. The Model

Our intent is to capture some key behavioral elements of an emerging market economy that eschews complete exchange-rate flexibility, monitors its international reserve holdings, and is imperfectly integrated into world financial markets. It also has influence over the price of its domestic output. We construct a two-period model of this open economy. The economy has a precautionary motive for acquiring and holding international reserves and accumulates reserves when it chooses a weak currency to achieve its output target. The reserves process is analyzed for two policy rules, discretion and a rule with escape clause.

Our framework draws on previous work by Clarida, Gali and Gertler (1999), Svensson (2000), Romer (2002) and Bar-Ilan and Lederman (2007), among others. The policymaker minimizes the following two-period loss function that includes both standard and non-standard elements:

$$L = \min\{E_1 \sum_{t=1}^{2} \rho^{t-1} [\frac{1}{2}(x_t^2 + \alpha\pi_t^2 + \beta(R_t - \hat{R})^2) + C\eta_t]\} \tag{1}$$

The variable x_t denotes the output gap – the deviation at time t of (log) output from (log) potential output, where the latter is normalized to zero. The variable π_t is the time t inflation rate, R_t is the time t log of international reserves, and $\hat{R}$ is the log of optimal reserves. The discount rate is ρ, while E_t represents expectations formed at period t. The parameters α and β are non-negative relative weights attached to deviations in prices and reserves, respectively. Equation (1) describes a policymaker focused on output-gap stabilization, domestic inflation stabilization, and international reserve targeting.

Two non-standard elements in the loss function are the reserve-targeting objective and the cost of a crisis. The policymaker is concerned about reserve targeting because extreme reserve levels —very low levels or very high levels-- are costly, as described above.[6]

The last term in the loss function captures the cost to the policymaker when the economy plunges into crisis. The parameter $C \geq 0$ is a fixed loss suffered by the policymaker if reserves fall below a critical value, triggering a crisis that includes a forced change in the exchange rate and damage to the policymaker's reputation and

[6]Some previous models have included a component of reserves or an international payments imbalance as an objective of policy. For example, Frankel (1988) considers a policymaker who wants to stabilize not only output and inflation but also the current account as a share of GDP. The ultimate argument for putting weight on inflation and the current-account deficit, according to Frankel, is that "society views these variables as "bads", and can be said to have a utility function that includes them….An economist who maximizes a theoretical welfare function that excludes such variables is not solving a problem to which society wants the answer." (p. 14). Isard (1994) considers a policymaker who is concerned with avoiding valuation losses on reserves as well as output stabilization. Jeanne and Svensson (2007) focus on a policymaker who is concerned not only with output stabilization and inflation targeting but the level of central bank capital, where the latter is related to the domestic-currency value of international reserves.

credibility.[7] The cost can also be interpreted as a deadweight loss to society as a whole. The parameter η is an indicator variable for this event. It takes the value of 0 or 1 depending on whether reserves are above the critical value needed to sustain the planned exchange rate policy and avoid a crisis:

$$\eta_t = \begin{cases} 0 & \text{no crisis} \\ 1 & \text{crisis} \end{cases}$$

For an economy with an exchange-rate peg, we assume the peg is maintained if there is sufficient reserve backing. The assumption implies a prohibitively high cost for reneging on the peg when reserves are above their critical value.[8]

The economy's aggregate demand and supply functions are:

$$x_t = \vartheta(s_t - p_t) + g_t \;; \qquad E(g_t) = 0, \quad Var(g_t) = \sigma_g^2 \tag{2}$$

$$\pi_t = \lambda x_t + \gamma \pi_{t-1} + u_t \;; \qquad E(u_t) = 0, \quad Var(u_t) = \sigma_u^2 \tag{3}$$

Aggregate demand for domestically produced goods is given by equation (2), an "IS" curve that relates the output gap to the real exchange rate, where s_t is the log of the

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[7]A first-period crisis does not affect the policymaker's behavior. The policymaker can respond *ex post* to a period 1 crisis by changing exchange-rate policy, but the policymaker can do nothing to avoid a period 1 crisis—it depends on inherited reserves R_0 and the realization of first-period shocks.

[8] For simplicity, we assume that *C* is the same for discretion and the peg. It could easily be the case that *C* is larger for reneging on a peg because the loss of reputation is greater. Imposing different costs would be easy to study, but we do not pursue it here.

nominal exchange rate (home currency price of foreign exchange), p_t is the log price of domestically-produced output, and the log foreign price has been normalized to zero. ϑ is a positive parameter and g_t is a real demand shock assumed to be white noise.[9]

Equation (3) is a traditional Phillips Curve (aggregate supply equation) that relates inflation positively to the output gap and past inflation. λ, γ, and ψ are positive parameters, with $0 \leq \gamma \leq 1$. The inflation shock, u_t, is also assumed to be white noise. Despite criticism that inflation is determined less by past inflation than by anticipated future inflation, the traditional Phillips Curve does a reasonable job of characterizing post-war inflation in the U.S. and in the Euro area.[10] Little is known about whether a traditional Phillips Curve or a "new" Phillips Curve with anticipated future inflation best fits the data for emerging markets. In future work, we plan to analyze the implications of alternative specifications. Currently, only our policymaker is forward-looking.[11]

Equation (4) describes the behavior of international reserves:

$$R_t = R_{t-1} + \theta(s_t - p_t) + v_t \quad ; \qquad E(v_t) = 0, \quad Var(v_t) = \sigma_v^2 \qquad (4)$$

θ is a positive parameter, so reserves are positively related to the real exchange rate. A more depreciated real exchange rate improves competitiveness and the current account, yielding a positive change in reserves. Capital flows are not modeled explicitly, but their

[9] In a future extension, we shall consider the implications of having disturbances follow AR1 processes. The parameter ϑ could be negative if a real depreciation that accompanies a crisis contracts output due to the balance-sheet problems of firms and banks.

[10] See Gali et al (2001).

[11] Both the demand and supply functions can be derived from micro-foundations.

influence on reserves is captured by v_t, the white noise disturbance to the reserves process that reflects shocks to the current account and financial account.

In order to maintain sufficient backing for the exchange rate, reserves cannot fall below a critical lower bound, denoted by B. Hence,

$$R_t \geq B, \qquad B \geq 0 \tag{5}$$

Finally, the (log) price of domestically-produced output is p_t, where

$$p_t = p_{t-1} + \pi_t \tag{6}$$

3. The Solution under Discretion

Under discretion, the policymaker chooses the nominal exchange rate each period in order to minimize the loss function (1) subject to the constraints (2)-(6). To start, we simplify the problem by assuming no reserve targeting (β=0 in the loss function) and no effect of past inflation on current inflation ($\gamma = 0$ in equation (3)). We then solve the problem recursively.

Period 2

Period 2 is the terminal period, so intertemporal considerations do not play a role. We first ignore the international reserves constraint (5) to solve for the *unconstrained* discretionary solution:

$$s_2 - p_2 = -\frac{g_2}{\vartheta} - \frac{\alpha\lambda}{\vartheta(1+\alpha\lambda^2)} u_2 \quad (7)$$

$$x_2 = -\frac{\alpha\lambda}{1+\alpha\lambda^2} u_2 \quad (8)$$

$$\pi_2 = \frac{1}{1+\alpha\lambda^2} u_2 \quad (9)$$

$$R_2 = R_1 - \frac{\theta}{\vartheta} g_2 - \frac{\theta\alpha\lambda}{\vartheta(1+\alpha\lambda^2)} u_2 + v_2 \quad (10)$$

The unconstrained discretionary solution is similar to Clarida et al (1999), though adapted here for an exchange-rate instrument rather than an interest-rate one. Recall that the exchange rate is selected after the realization of second-period shocks. The policymaker sets the nominal exchange rate in order to obtain the desired real exchange rate in (7).

The unconstrained discretionary solution for the second period shows the impact of period two shocks and the state variable R_1. For example, we see from (7)-(10) that the real shock, g_2, is offset by a policy of currency appreciation and does not affect output or inflation. It does lead to reduced reserves in the second period because the currency appreciation worsens the current account. A positive nominal shock, u_2, also leads the policymaker to appreciate the currency, but now the effects of the inflationary shock are split optimally between reduced output and increased inflation. Second-period reserves decline in response to the currency appreciation.[12]

We next take into account the reserves constraint. Suppose the discretionary solution for the real exchange rate gives insufficient reserves to satisfy the reserves constraint. Specifically, substituting (7) into (4) and (4) into (5), the policymaker observes a violation of the reserves constraint when:

$$v_2 - \frac{\theta}{\vartheta} g_2 - \frac{\alpha\lambda\theta}{\vartheta(1+\alpha\lambda^2)} u_2 < B - R_1 \qquad (11)$$

Equation (11) shows that a violation of the reserves constraint is more likely to occur if the economy inherits low reserves from period one, faces negative shocks to the reserves process, or experiences large positive shocks to output or inflation that appreciate the real exchange rate and erode the reserve position.

Should reserves fall below their critical value, it triggers a crisis. The crisis forces the policymaker to accept a more depreciated value for the exchange rate to build up reserves, damages the policymaker's reputation, and pushes the economy away from its optimal output and inflation targets. Let us examine each of these consequences in turn.

If the discretionary solution for the exchange rate violates the reserves constraint, the policymaker must depreciate the nominal exchange rate until $R_2 = B$. From (4) and (5), the constrained exchange-rate choice must be:

[12] The fact that the reserves shock affects output or inflation with a one-period lag is an artifact of the model specification. If foreign real and nominal shocks that affect the reserves process via the shock v also affected the IS curve or Phillips Curve, they would also influence output and inflation contemporaneously.

$$s_2 - p_2 = \frac{1}{\theta}(B - R_1 - v_2) \tag{12}$$

The expected cost of the crisis to the policymaker is:

$$C * \Pr[v_2 - \frac{\theta}{\vartheta} g_2 - \frac{\alpha\lambda\theta}{\vartheta(1+\alpha\lambda^2)} u_2 < B - R_1] \tag{13}$$

where $C \geq 0$ is the fixed cost of the crisis to the policymaker in terms of lost reputation and Pr[.] is the probability a crisis occurs because the reserve constraint is violated, meaning the inequality in (11) holds.

The crisis is also costly to the economy. The required adjustment in exchange-rate policy induced by the crisis means that output and inflation cannot be at their first-best, unconstrained levels. In order to evaluate this cost, note from equations (2) and (3) that when the real exchange rate changes by an increment δ_t, the output gap and inflation change by:

$$\Delta x_t = \vartheta \delta_t \tag{14}$$

$$\Delta \pi_t = \lambda \Delta x_t = \lambda \vartheta \delta_t \tag{15}$$

Using a second-order Taylor approximation, the welfare cost in period 2 of changing the real exchange rate by δ_2 relative to its unconstrained value is:

$$\Delta L_2(R_1) = \frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\delta_2^2 \qquad (16)$$

The expected increase in the period two loss because of the deviation of output and inflation from their unconstrained levels is:

$$E\Delta L_2(R_1) = \frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\int_{\delta_2=0}^{\infty}\delta_2^2 f(\delta_2)d\delta_2 \qquad (17)$$

The integral in (17) is over the values of the second-period shocks that satisfy the inequality (11), and δ_2 $(0 \le \delta_2 < \infty)$ is the required increase in the real exchange rate to satisfy the reserves constraint. That is, subtracting (7) from (12):

$$\delta_2 = \frac{1}{\theta}(B - R_1 - v_2) + \frac{g_2}{\vartheta} + \frac{\alpha\lambda u_2}{\vartheta(1+\alpha\lambda^2)} \qquad (18)$$

The total additional expected cost of moving to the constrained exchange-rate policy relative to the unconstrained policy is the sum of (13) and (17). This additional cost of a crisis is lower when the economy enters period 2 with greater inherited reserves, R_1. Consequently, a greater stockpile of reserves not only reduces the probability of a crisis but lowers its economic costs should one occur. These benefits are important features of the insurance story of reserve accumulation.

Period 1

Period 1 starts with an exogenous amount of reserves, R_0. Then the shocks are realized and the policymaker chooses the exchange rate. The policymaker faces a trade-off, however. If the policymaker adopts a weak currency, the economy gains reserves R_1 that reduce the expected loss in the second period from having insufficient reserves, a crisis, and constrained exchange-rate policy. On the other hand, adoption of a weak currency may not give optimal first-period output and inflation.

Suppose the policymaker is not forward-looking. Then the policymaker will not take into account the chance of a period 2 crisis. The policymaker's problem is then similar to the one described previously for period 2. However, the solution that was optimal for the terminal period is myopic and sub-optimal in period 1. The myopic real exchange rate is:

$$(s_1 - p_1)_{myopic} = \max[-\frac{g_1}{\vartheta} - \frac{\alpha\lambda}{\vartheta(1+\alpha\lambda^2)}u_1,\ \ \frac{1}{\theta}(B - R_0 - v_1)] \quad (19)$$

Equation (19) says the myopic policymaker chooses the unconstrained solution in period one unless the reserves constraint is violated, in which case the constrained solution is chosen.

Define δ_1 as the difference between the real exchange that takes into account intertemporal concerns, $s_1 - p_1$, and the myopic real exchange rate that does not:

$$\delta_1 = (s_1 - p_1) - (s_1 - p_1)_{myopic} \quad (20)$$

A forward-looking policy maker takes into account the possibility of a period 2 crisis and hence chooses an exchange rate in period 1 that helps ensure sufficient future reserves. Consequently, period 1 output and inflation cannot be at values unconstrained by period 2 concerns, and the period 1 welfare loss is:

$$\Delta L_1(R_0) = \frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\delta_1^2 \qquad (21)$$

The discretionary real exchange rate solution for period one, $s_1 - p_1$, is chosen by a forward-looking policymaker to minimize the net present value of the welfare loss given by equations (13), (17) and (21). Specifically, the discretionary outcome is the solution to:

$$\min_{\delta_1} \rho\{C * \Pr[v_2 - \frac{\theta}{\vartheta}g_2 - \frac{\alpha\lambda\theta}{\vartheta(1+\alpha\lambda^2)}u_2 < B - R_1] + \frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\int_{\delta_2=0}^{\infty}\delta_2^2 f(\delta_2)d\delta_2\}$$
$$+\frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\delta_1^2 \qquad (22)$$

where δ_2 is defined in (18) and $s_1 - p_1$ is given by substituting δ_1 into (20) and using (19). We discuss the properties of this solution in Section 5.

4. The Solution under a Rule

Under a rule, the policymaker chooses the nominal exchange rate in the first period in order to minimize the loss function (1) subject to the constraints (2)-(6). Again, we simplify by assuming no reserve targeting (β =0 in the loss function) or effect of past inflation on current inflation ($\gamma = 0$ in equation (3)). We solve this problem recursively.

Period 2

Period 2 starts with two state variables, reserves, R_1, and the nominal peg, $\bar{s}$, set in the previous period.[13] The shocks (g_2, u_2, v_2) are drawn. If the resulting reserves are equal to or above the critical value B, the rule (peg) is kept; otherwise, the peg is abandoned. Suppose the peg is kept. Then taking into account that $p_2 = p_1 + \pi_2 = p_0 + \pi_1 + \pi_2 = \pi_1 + \pi_2$, where we assume $p_0 = 0$, and making further substitutions, we get the period 2 solution under the rule:

$$\bar{s} - p_2 = \frac{\bar{s} - \pi_1 - \lambda g_2 - u_2}{1 + \vartheta\lambda} \tag{23}$$

$$x_2 = \frac{\vartheta(\bar{s} - \pi_1) - \vartheta u_2 + g_2}{1 + \vartheta\lambda} \tag{24}$$

$$\pi_2 = \frac{\lambda\vartheta(\bar{s} - \pi_1) + \lambda g_2 + u_2}{1 + \vartheta\lambda} \tag{25}$$

$$R_2 = R_1 + \frac{\theta(\bar{s} - \pi_1) - \theta\lambda g_2 - \theta u_2}{1 + \vartheta\lambda} + v_2 \tag{26}$$

[13] Under discretion, the only state variable is reserves R_1. This difference will play an important role in the results presented in the next section.

Under the rule, both real and nominal shocks affect second-period output and inflation.[14] The real exchange rate appreciates in response to positive values of both shocks, but only because prices rise, since the nominal exchange rate is pegged. Under discretion, the real exchange rate appreciated because the policymaker lowered the nominal exchange rate. Both real and nominal shocks affect second-period reserves because they alter the real exchange rate.

Suppose now the peg is unsustainable in period 2 because it yields reserves below their critical value:

$$v_2 - \frac{\theta(u_2 + \lambda g_2)}{1 + \lambda \vartheta} < B - R_1 - \frac{\theta(\bar{s} - \pi_1)}{1 + \lambda \vartheta} \tag{27}$$

According to equation (27), reserves may not be adequate to maintain the peg if there are big, positive shocks to output or inflation, big negative shocks to the balance of payments, or the peg gives too strong a currency.

However, even if there are sufficient reserves to keep the peg, second-period output and inflation need not be optimal as they would be under discretion. Letting $\delta_{2,1}$ denote the difference between the real exchange rate under the rule (23) and the real exchange rate under discretion (7), we note that:

$$\delta_{2,1} = \begin{cases} 0 & \text{when (27) is satisfied} \\ \dfrac{\bar{s} - \pi_1 - \lambda g_2 - u_2}{1 + \lambda \vartheta} + \dfrac{g_2}{\vartheta} + \dfrac{\alpha \lambda u_2}{\vartheta(1 + \alpha \lambda^2)} & \text{otherwise} \end{cases} \tag{28}$$

[14] This outcome differs from discretion, where the real shock had no impact on output or inflation.

According to (28), if there are insufficient reserves, the peg is abandoned and there is no welfare loss from having stayed on the peg rather than moving to discretion. If there are sufficient reserves, the peg is kept and the expected loss of keeping the peg relative to the first-best discretionary outcome is:

$$\frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_{-\infty}^{\infty}\delta_{2,1}^2 f(\delta_{2,1})d\delta_{2,1} \tag{29}$$

Having insufficient reserves under the peg triggers a crisis. The expected cost to the policymaker of reneging on the peg in a crisis is:

$$C*\Pr[v_2-\frac{\theta(u_2+\lambda g_2)}{1+\lambda\vartheta}<B-R_1-\frac{\theta(\bar{s}-\pi_1)}{1+\lambda\vartheta}] \tag{30}$$

where we make use of (27). The policymaker forced to renege on a peg in a crisis can move to the discretionary solution (7)-(10) if this solution yield sufficient reserves. Otherwise, the policymaker is constrained to set the exchange rate so reserves equal the critical lower bound, B . Consequently, if the peg is abandoned, the new real exchange rate is:

$$s_2-p_2=\max\{-\frac{g_2}{\vartheta}-\frac{\alpha\lambda u_2}{\vartheta(1+\alpha\lambda^2)},\ \frac{1}{\theta}(B-R_1-v_2)\} \tag{31}$$

Denote $\delta_{2,2}$ as the difference between the real exchange rate chosen when the policymaker is constrained and the real exchange rate chosen under discretion. That is,[15]

$$\delta_{2,2} = \frac{1}{\theta}(B - R_1 - v_2) + \frac{g_2}{\vartheta} + \frac{\alpha\lambda u_2}{\vartheta(1+\alpha\lambda^2)} \tag{32}$$

The expected loss from having to select the constrained real exchange rate is:

$$\frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_0^\infty \delta_{2,2}^2 f(\delta_{2,2})d\delta_{2,2} * \Pr[v_2 - \frac{\theta(u_2+\lambda g_2)}{1+\lambda\vartheta} < B - R_1 - \frac{\theta(\bar{s}-\pi_1)}{1+\lambda\vartheta}] \tag{33}$$

Summing equations (29), (30) and (33) gives the total expected loss in period 2 when the peg is adopted in period 1,

$$\frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_{-\infty}^\infty \delta_{2,1}^2 f(\delta_{2,1})d\delta_{2,1} \quad +$$

$$[C + \frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_0^\infty \delta_{2,2}^2 f(\delta_{2,2})d\delta_{2,2}] * \Pr[v_2 - \frac{\theta(u_2+\lambda g_2)}{1+\lambda\vartheta} \tag{34}$$

$$< \quad B - R_1 - \frac{\theta(\bar{s}-\pi_1)}{1+\lambda\vartheta}]$$

The first term in (34) denotes the loss from keeping the peg and having second-period output and inflation deviate from their first-best, discretionary levels. The second term denotes the loss when the peg is abandoned in the second period. This loss is composed

[15] Equation (32) is identical to (18), but the interpretation is different. Under discretion, a crisis always meant (18) was larger than zero. That is, the second period discretionary solution delivered insufficient reserves, so the real exchange rate was the constrained one. Under the rule, when the peg yields insufficient reserves, the peg is abandoned to discretion, with no additional loss, if discretion gives sufficient reserves; otherwise, the peg is abandoned to the constrained one, when (32) is positive.

of the fixed cost C and the output and inflation loss if the peg cannot be abandoned to discretion but instead the real exchange rate must be set at its constrained level.

Period 1

Period 1 starts with inherited reserves, R_0, and an inherited price of domestically-produced output normalized to zero, $p_0 = 0$. First-period shocks occur and a decision about the pegged exchange rate is taken. If the policymaker is myopic rather than forward-looking, the peg will be set so the first-period real exchange rate, $\bar{s} - p_1$ is either the discretionary one, (7), if this rate yields sufficient reserves, or the constrained real exchange rate, the one that produces reserves equal to the critical value, B. That is, $(\bar{s} - p_1)_{myopic}$ is given by (19).

Define δ_1 as the difference between the real exchange rate when the nominal peg is chosen by a forward-looking policymaker and the real exchange rate when the peg is set by the myopic policymaker. The incremental welfare loss in period 1 of having the peg determined by a forward-looking policymaker instead of a myopic one is given by (21). The peg is determined by solving the following problem:

$$\min_{\delta_1} \rho\{[C + \frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_0^{\infty} \delta_{2,2}^2 f(\delta_{2,2}) d\delta_{2,2}] * \Pr[v_2 - \frac{\theta(u_2 + \lambda g_2)}{1+\lambda\vartheta} < B - R_1 - \frac{\theta(\bar{s} - \pi_1)}{1+\lambda\vartheta}] +$$

$$\frac{\vartheta^2}{2}(1+\alpha\lambda^2)\int_{-\infty}^{\infty} \delta_{2,1}^2 f(\delta_{2,1}) d\delta_{2,1}\} + \frac{1}{2}\vartheta^2(1+\alpha\lambda^2)\delta_1^2 \qquad (35)$$

where $\delta_{2,1}$ is defined in (28) and $\delta_{2,2}$ is defined in (32). From (19) and (20), the real exchange rate is $\bar{s} - p_1 = (s_1 - p_1)_{myopic} + \delta_1$. Given the choice of the real peg, we can substitute $\bar{s} - p_1$ into equations (2)-(4) to obtain solutions for period 1 output, inflation and reserves. To find the nominal peg, the forward-looking policymaker solves

$$\bar{s} = (\bar{s} - p_1) + p_1 = (1 + \frac{1}{\gamma\vartheta}) p_1 - \frac{g_1}{\vartheta} - \frac{u_1}{\lambda\vartheta}.$$

5. A Comparison of Discretion and the Rule with Escape Clause

Using simulation methods, we numerically solve for the optimal difference between the first-period real exchange rate chosen by a forward-looking policymaker and the rate chosen by the myopic policymaker. We do so for both discretion and the rule with escape clause. Recall that this optimal difference is defined as δ_1. We also compare first-period real and nominal exchange rates under discretion and the rule, and we examine reserves and the welfare loss under the two regimes.

The solutions we present correspond to the case where the policymaker tries to stabilize output and inflation but is unconcerned with reserves stabilization. In the next section, we see what difference it makes when reserves stabilization is also an objective of policy.

We adopt the following solution strategy. For discretion, we choose the initial level of reserves (R_0) and the realization of the three first-period shocks. We then randomly select 100,000 values of second-period shocks drawn from a jointly normal density function with zero mean, variances $\sigma_g^2, \sigma_u^2, \sigma_v^2$, and zero cross-correlations.[16] For each set of second-period shocks, we obtain the loss that corresponds to a given δ_1. Recall that δ_1 determines the real exchange rate in the first period, $s_1 - p_1$.[17] We compute the average loss for all draws to obtain the expected loss associated with that δ_1 (that $s_1 - p_1$). We then choose another value for δ_1 (that is, $s_1 - p_1$) and obtain the expected loss associated with it. We repeat the exercise on small increments of $s_1 - p_1$ and finally get the optimal first-period real exchange that minimizes the expected loss (22).[18] For the rule, we follow the same strategy except that $\bar{s} - p_1$ determines both first-period reserves, R_1, and the nominal exchange rate peg for period two.

Since exchange rates, international reserves, and the welfare loss are functions of the same parameter values under discretion and the rule, we can compare them under the two regimes. Figures 1-5 plot these key variables as a function of initial reserves.

Figure 1 shows the optimal δ_1 under discretion and the rule. Recall that δ_1 measures the extent to which the real exchange rate chosen by a forward-looking policymaker in period 1 deviates from the real exchange chosen by the myopic policymaker.

Under discretion, when initial reserves are very high, the myopic policymaker sets the exchange rate at its discretionary value to achieve output and inflation stabilization in

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[16] Parameter values are $B = 5, \vartheta = 80, \lambda = 0.01, \theta = 100, \alpha = 2, \sigma_g = 1, \sigma_u = 0.01,$ $\sigma_v = 0.1, C = 200, p_0 = 0, \rho = 0.95.$ The three first-period shocks, known when the policy is determined, are set at zero. Initially we set $\beta = \gamma = 0.$

[17] See Equation (20).

[18] The loss function resembles a parabola with a well-defined minimum.

period 1. The forward-looking policymaker chooses the same exchange rate (so δ_1 =0), since large initial reserves minimize the chance of a future crisis. Making the currency weaker than the myopic value to accumulate additional reserves does not reduce the probability of a future crisis very much but is costly in terms of output and inflation deviations from their first-best, unconstrained values.

Since the myopic and forward-looking policymakers both choose the same real exchange rate when initial reserves are very high (Figure 3), they both obtain the same value of reserves in period 1, as illustrated in Figure 2. We define precautionary reserves as the additional reserves acquired in period 1 when the policymaker is forward-looking rather than myopic. Since $R_1^{myopic} = R_1^{forward-looking}$, precautionary reserves are zero when the economy starts out with very large reserves.

What happens when initial reserves are extremely low? Under discretion, the myopic policymaker chooses the discretionary real exchange rate solution, if it raises reserves to B; otherwise, the policymaker chooses the constrained exchange-rate solution to bring reserves up to B. The forward-looking policymaker recognizes that R_1 also affects the chance of a future crisis and therefore will adopt an even weaker currency than the myopic policymaker in order to accumulate reserves in excess of B. Figures 1 and 2 illustrate that when initial reserves are very low, δ_1 is positive (the real exchange rate is weaker than the myopic one) and reserves in period 1 exceed B, the myopic target. Indeed, the simulation shows that these reserves contain a substantial precautionary component.[19]

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[19] Since we set $B = 5$ in the simulation, when $(R_0 - B)/B = -1$, $R_0 = 0$ and the forward-looking policymaker accumulates $R_1 = 8.2$. These reserves are at least 60% higher than the reserves acquired by

What happens when initial reserves are in the mid-range of Figures 1-2? When $R_0 < B$, the myopic policymaker chooses an exchange rate to bring first-period reserves to B. The forward-looking policymaker always chooses a weaker currency than the myopic policymaker ($\delta_1 > 0$) in order to accumulate more reserves and reduce the chance of a future crisis. When $R_0 > B$, the myopic policymaker chooses the discretionary real exchange rate solution. Because higher R_0 provides greater protection against a future crisis, the forward-looking policymaker can choose a currency value closer to the myopic one-- with an eye towards stabilizing output and inflation-- and not be so concerned with accumulating additional reserves. Consequently, for $R_0 > B$, an increase in R_0 reduces δ_1 and precautionary reserves, while R_1 increases with R_0.

What is the story under the peg? When reserves are very high, the story is similar to the one for discretion. The forward-looking policymaker chooses the same exchange rate as a myopic one (so δ_1 =0) since reserves are already plentiful and can minimize the chance of a future crisis. Precautionary reserves are zero.

When reserves are very low, things are different. As before, the myopic policymaker chooses a peg (either the discretionary or constrained solution) to increase reserves to $R_1 = B$. But in contrast to the case with discretion, the forward-looking policymaker operating a peg has no incentive to adopt an even weaker currency peg to accumulate more reserves and reduce the chance of a future crisis. (Hence δ_1 =0.) The reason is that reserves are no longer the only defense against a future crisis. A weak-currency peg carried over to the next period also reduces the chance of a future crisis.

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the myopic policymaker, who sets $R_1 = B = 5$. Recall that reserves over and above those acquired by the myopic policymaker are precautionary reserves.

Figures 1-3 show that the more initial reserves fall below their critical value, the weaker the currency peg under myopic policy and hence the smaller is δ_1. Since the weak currency peg is carried over to the next period, reserves need not take on the full defense burden.

Figures 3 and 4 show the optimal real and nominal exchange rates selected in period one under discretion and the rule. For the rule, a lower R_0 requires a higher $\bar{s} - p_1$ and $\bar{s}$ (a weaker currency) in order to build up additional reserves needed to meet the required level B. The same is true for discretion, but the policymaker must choose to make the currency even weaker in the first period since the weak currency is not carried over to next period. Consequently, as Figure 2 shows, reserves in period one are higher under discretion than under the rule. Under discretion, higher reserves are the defense against future crises. Under the rule, the defense is twofold. There is some accumulation of reserves in period one (though less than under discretion) and there is the adoption of a weak-currency peg that carries over to the next period. The weak-currency peg substitutes for aggressive reserve accumulation.

Figure 5 illustrates the net present value of the welfare loss under discretion and the rule as a function of initial reserves. For most values of initial reserves, the welfare loss is smaller under the peg than under discretion. How can it be that constraining exchange-rate policy is often welfare-enhancing? In the standard closed-economy story, constraining the central bank to follow a rule can be welfare-improving because it reduces inflationary bias. In our story, constraining policy to adopt a peg that will be maintained in the future means the currency need not be so weak in the first period, allowing output and inflation to get closer to their first-best values in period one. Under discretion, the policymaker must maintain a weaker currency in the first period to obtain higher first-period reserves, since reserves are the only defense against a future crisis. Under the rule, both first-period reserves and the value of the peg influence the probability of a future crisis. This commitment to the peg is particularly important during a crisis, so the additional welfare loss of discretion relative to the rule increases as R_0 decreases.

6. Extensions

6.1 Output Shocks

We now examine how exchange rates, reserves and the welfare loss are affected by the output shock g_1. Figures 6-10 illustrate this case.

Suppose initial reserves are at their lower bound ($R_0 = B = 5$) and there is a large negative output shock. The economy finds itself with output below potential, a situation not too different from China's over much of the past fifteen years.

What is the optimal response to the bad output shock? The myopic policymaker depreciates the nominal exchange rate (and the real exchange rate also) with the objective of stabilizing output and inflation. The myopic policymaker chooses the discretionary solution since initial reserves are at *B*. The forward-looking policymaker chooses the same exchange rate as the myopic one (δ_1=0 in Figure 6) since that exchange rate helps achieve output-inflation stabilization while generating first-period reserves above their initial value to give protection against a future crisis. The sizeable reserves accumulated in the first period (Figure 7) have no precautionary component--they are sizeable whether the policymaker is myopic or forward-looking. For a country like China, where output was probably much below potential for an extended period, optimal behavior under discretion requires the adoption of a very weak currency in both real and nominal terms (Figures 8 and 9). As a result, the economy accumulates substantial reserves in period 1.

The story is the same under the rule. When output is significantly below potential (when there are very bad output shocks), it is optimal to choose the same δ_1, $\bar{s}$ and $\bar{s} - p_1$, and R_1 under the rule as under discretion, as shown in Figures 6-9. A country such as China finds it optimal to keep the currency weak and accumulate significant reserves whether it follows discretion or a rule.

The welfare losses from output shocks under discretion and the rule are depicted in Figure 10. When g_1 is negative but close to zero, commitment to the peg is preferable. It allows the forward-looking policymaker to set the real exchange rate closer to the myopic one[20] and to move first-period inflation and output closer to their optimal values. However, as g_1 gets more negative, the welfare loss becomes higher for the peg than for discretion. That is because the weak-currency policy adopted in period 1 must be carried over to period 2 under a peg, causing output and inflation to deviate more from their first-best values in period 2.[21] Consequently, discretion is preferable to the rule when output is significantly below potential.[22]

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[20] Figure 6 shows δ_1 is smaller under the rule for small negative output shocks.

[21] If g_1 follows an AR(1) process with a positive AR coefficient, the peg is not so bad even for very negative output shocks. Positively correlated output shocks imply the recession will last more than one period, so commitment to a weak currency both now and in the future makes more sense.

[22] What happens when there are big positive shocks to output? Barring any policy response, output will exceed the natural rate and inflation will increase. The myopic policymaker would like to appreciate the currency to achieve output and inflation stabilization, but is bound by the reserves constraint. An appreciation would reduce reserves below B, their initial value. Since the positive output shock generates some real appreciation via inflation, the myopic policymaker ends up engineering a nominal depreciation to

6.2 Reserves Dynamics

It would be useful to understand the dynamic paths of reserves under discretion and the rule and compare how closely those paths correspond to actual ones for Asian economies. Unfortunately, we are limited by a model that has only two periods. We did compute expected reserves in the second period, $E(R_2)$,[23] so that, along with optimal first-period reserves, R_1, we could learn something about the dynamics. We find that, for discretion, expected second-period reserves are very close to first-period reserves for our parameter values. For the peg, Figure 11 shows that for low levels of initial reserves, R_0, expected second-period reserves can be more than 30% higher than first-period reserves. This result is consistent with Figure 2. When R_0 is low, R_1 is much lower for the peg than for discretion. But since the weak-currency peg in period 1 is carried over to period 2 , expected reserves are higher in that period. The outcome demonstrates once again how a weak-currency peg can substitute for reserves in protecting against a future crisis.

6.3 Reserves Stabilization

The results discussed so far correspond to the case where the policymaker is concerned only with stabilizing output and inflation. We next examine the case where

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keep the real exchange rate unchanged. Consequently, first-period reserves stay constant no matter the size of the positive output shock (Figure 7), and the reserves constraint prevents the myopic policymaker from achieving first-best output and inflation in period 1. For positive output shocks, there is a greater real depreciation under discretion since reserves are the only defense against a future crisis and must be accumulated in period 1 (Figure 8). However, that makes it more difficult to achieve first-period output and inflation stabilization under discretion. Not surprisingly, the welfare loss is greater under discretion in the face of positive output shocks (Figure 10).

[23] $E(R_2)$ is based on realizations of second-period shocks that determine also whether the second-period reserves constraint is binding.

reserves stabilization is an additional objective of policy. We are interested in learning what difference it makes for exchange rates, reserves and the welfare loss to have this added policy objective. [Note: this case will be developed in the next draft.]

7. Conclusion

This paper nests the buffer stock model within a standard open-economy model. It investigates the behavior of international reserves and other macroeconomic variables when the policymaker pursues output and inflation stabilization and recognizes the linkages between exchange-rate policy and international reserves accumulation. The policymaker understands that the supply of reserves can constrain the choice of exchange rate and the choice of exchange rate affects the supply of reserves. The policymaker also knows that having too few reserves can increase the probability of a future crisis and the output and inflation costs associated with a crisis.

A few key insights are worth re-emphasizing. First, a peg can substitute for reserves in protecting against a future crisis. If the economy starts with few reserves, a policymaker needs a weak currency to accumulate reserves and reduce the chance of a future crisis. If the chosen exchange rate is anticipated to remain the same in the future, the policymaker need not make the currency as weak nor reserves as high in the current period. The commitment to the peg helps reduce the chance of a future crisis and does not place the full burden of defense on reserves.

Second, if the economy starts with few reserves, the peg delivers a smaller welfare loss than discretion. That is because the policymaker can rely on the peg as well as reserves to defend against a future crisis and so is better able to achieve output and inflation stabilization in the initial period. If the economy has high reserves, discretion is preferred because it is less constrained than a peg.

Third, when output is below potential, it is optimal under both discretion and the rule (a peg) to adopt a weak currency and promote export-led growth to achieve output and inflation stabilization. This policy also leads to reserve accumulation and is consistent with the behavior of China.

Fourth, even in theory, it is difficult to separate the insurance motive for reserve accumulation from the output motive for reserve accumulation. In our model, the policymaker has the incentive to reduce the chance of a future crisis and the output and inflation costs associated with a crisis, and so will select an exchange rate and its associated reserve accumulation with those considerations in mind. In addition, the policymaker is always pursuing output and inflation stabilization, optimally choosing the exchange rate, and obtaining reserves as a byproduct of these objectives. It makes little sense to compute the reserves needed to fulfill the insurance motive-- as in the buffer stock model-- while ignoring the broader policy objectives of output and inflation stabilization.

Further research needs to be undertaken to extend the model and learn whether the results described here are robust to these extensions. For example, it would be useful to modify the Phillips Curve specification so that inflation depends on expected future inflation. The IS curve could also reflect the fact that expectations of future exchange-rate policy affect private-sector spending. Forward-looking expectations of the private sector would then interact with forward-looking behavior of the policymaker to generate interesting outcomes for exchange-rate policy and reserve accumulation.

Another extension would be to model a cost for holding reserves when reserves stabilization is not an explicit objective of policy. At present, the cost of holding excessive reserves only comes to the forefront when reserves stabilization is in the loss function.

It would also be useful to explore the implications for exchange-rate policy, reserves, and welfare of having persistence in the macroeconomic shocks hitting the economy. Another extension would be to examine results when the policymaker can renege on a peg at a cost when reserves are above their lower bound. Finally, another important extension would be to specify an infinite-horizon version of the current two-period model in order to study more carefully the dynamic path of international reserves under discretion and the rule.

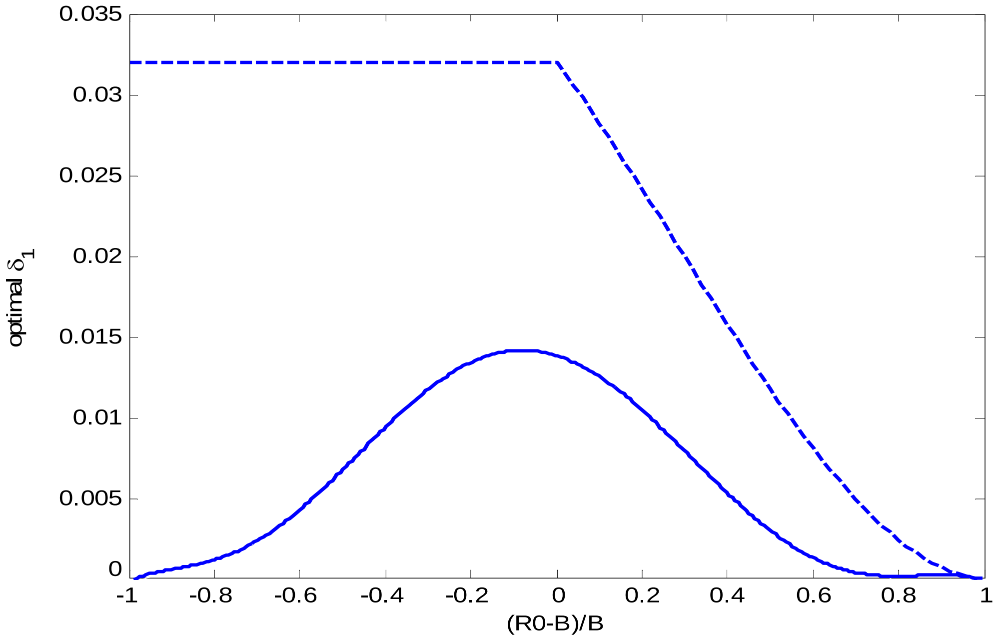

Figure 1. Real Exchange Rate in Excess of the Myopic Rate
Dashed line – discretion; Solid line - rule

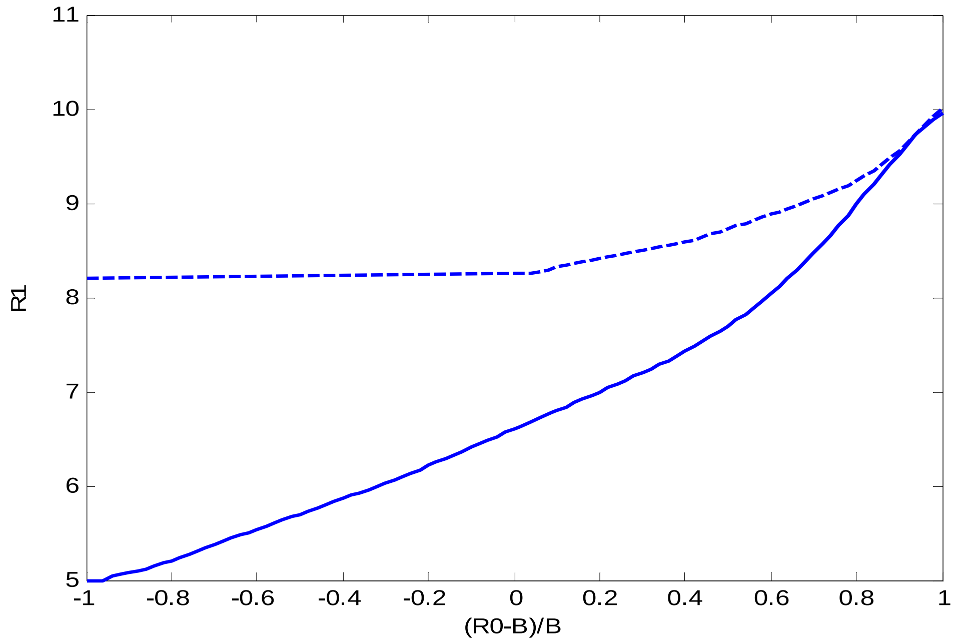

Figure 2. Optimal Reserves in the First Period
Dashed line – discretion; Solid line - rule

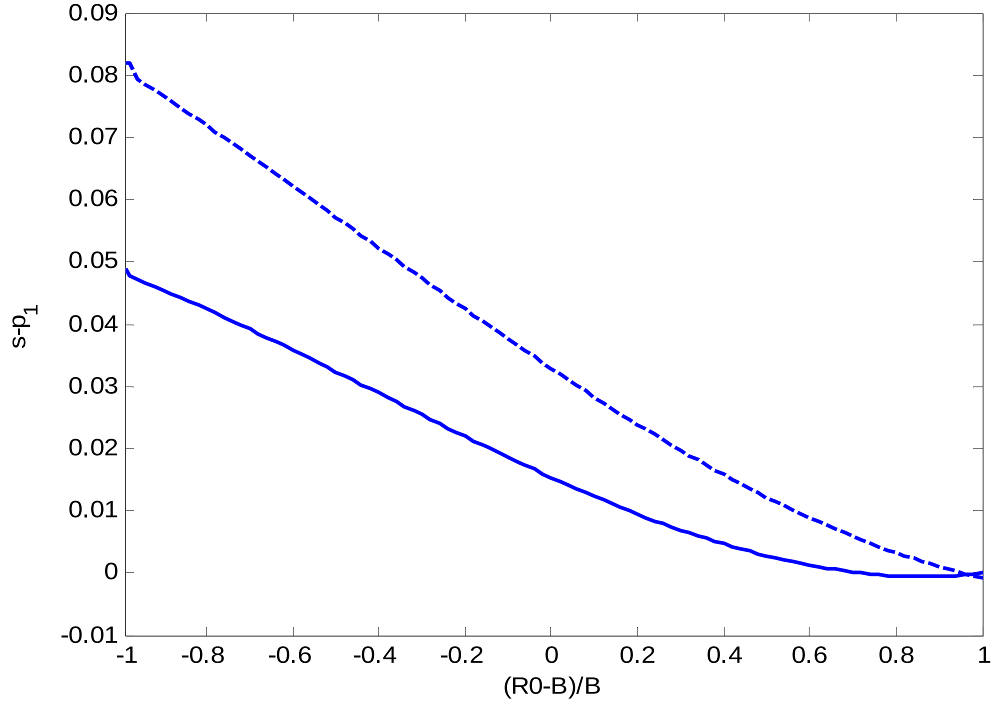

Figure 3. Optimal Real Exchange Rate in the First Period
Dashed line – discretion; Solid line - rule

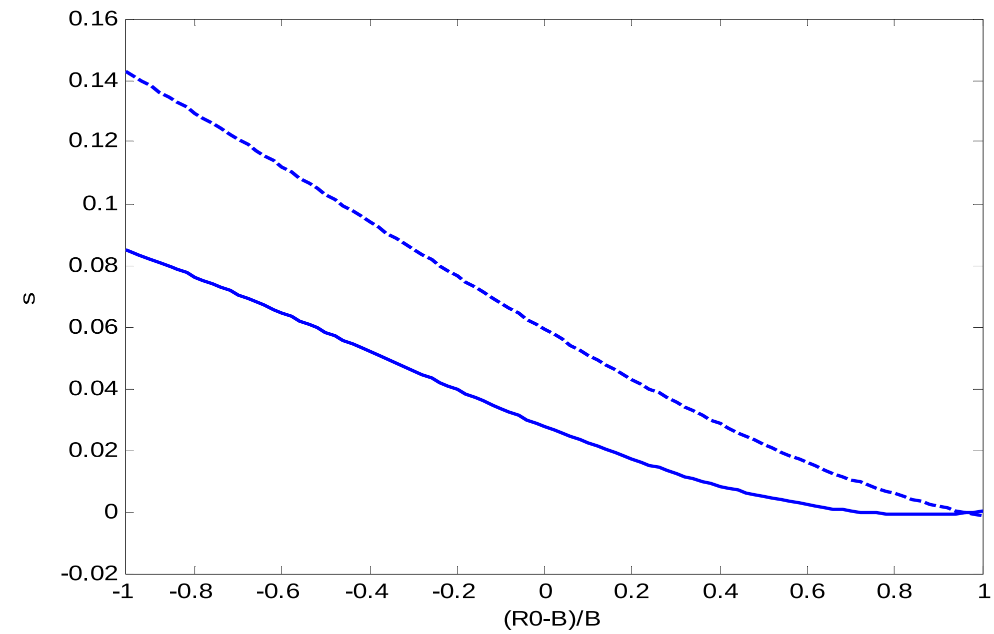

Figure 4. Optimal Nominal Exchange Rate
Dashed line – discretion; Solid line - rule

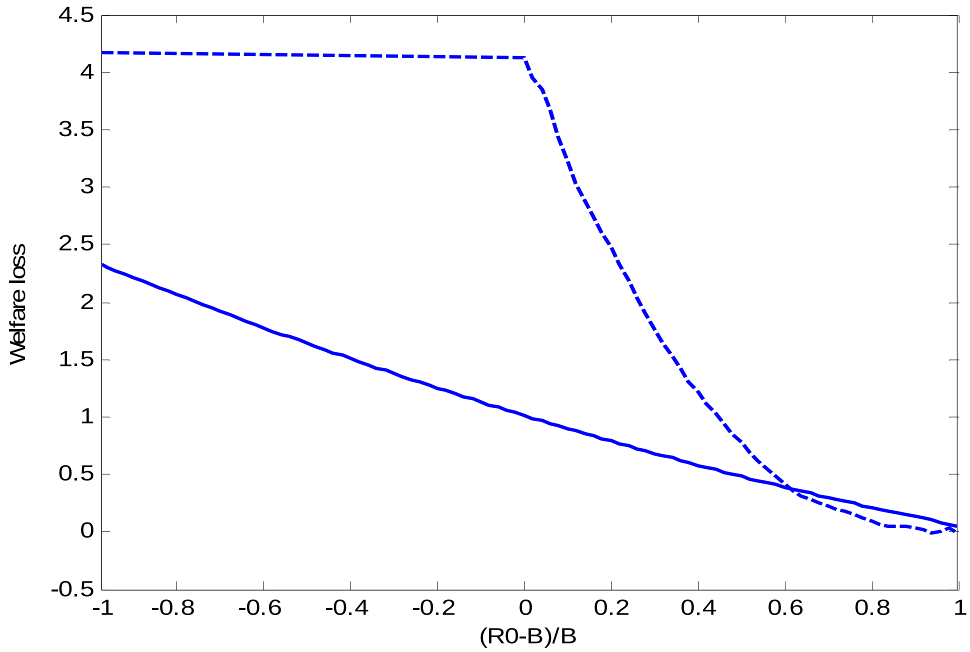

Figure 5. Welfare Loss
Dashed line – discretion; Solid line - rule

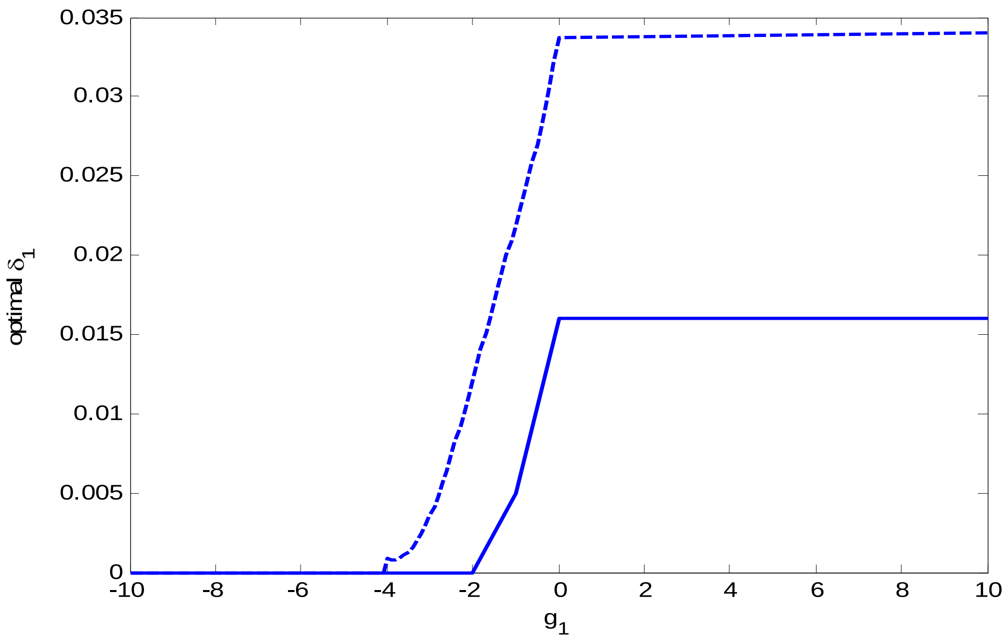

Figure 6. Real Exchange Rate in Excess of Myopic One with Output Shock
Dashed line-discretion; solid line-rule

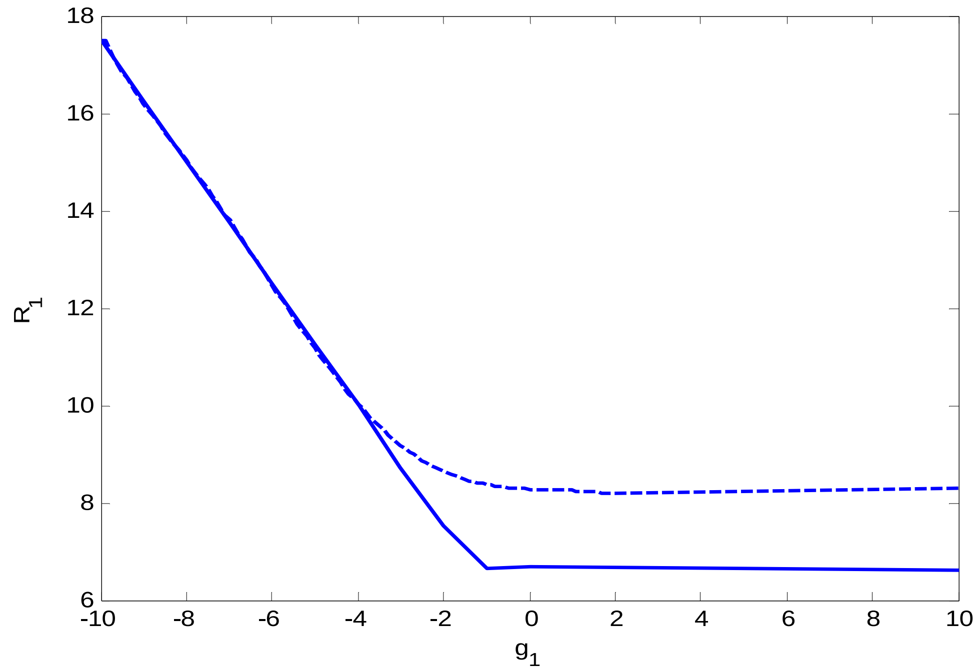

Figure 7. Reserves with Output Shock
Dashed line-discretion; Solid line-rule

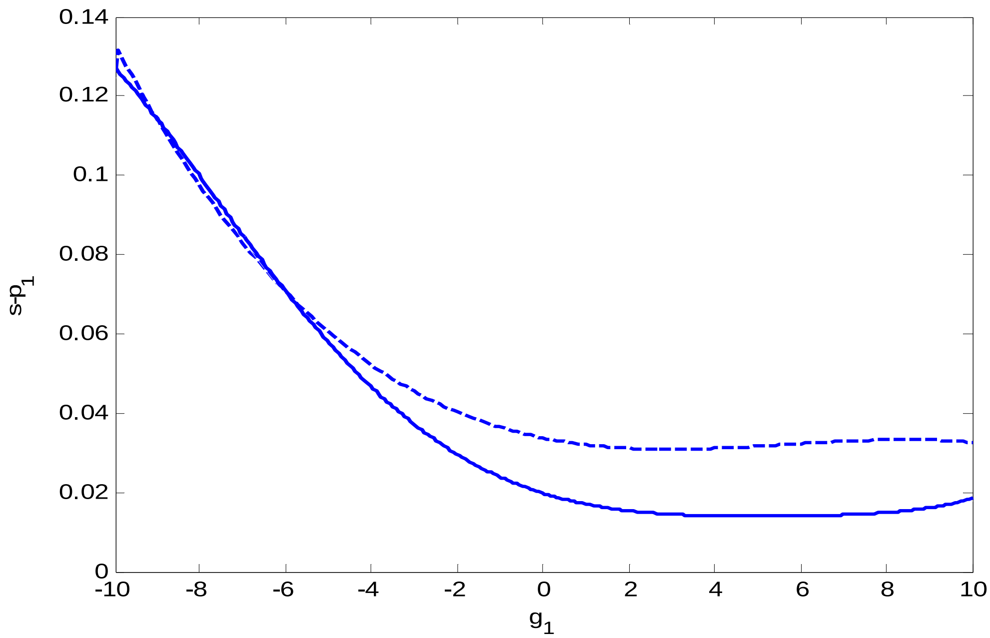

Figure 8. Real Exchange Rate with Output Shock
Dashed line-discretion; solid line-rule

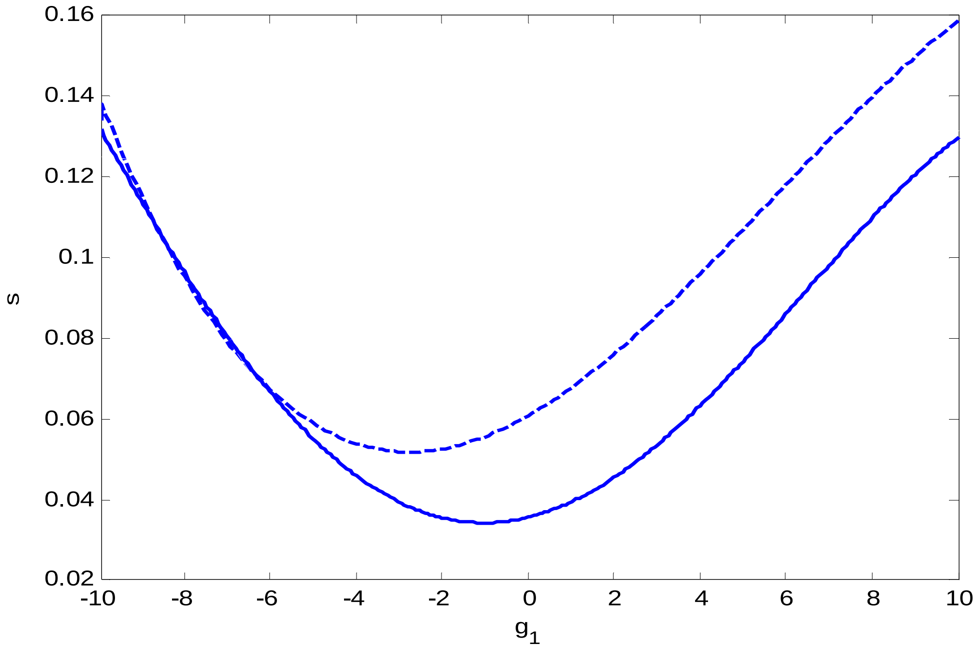

Figure9. Nominal Exchange Rate with Output Shock
Dashed line-discretion; solid line-rule

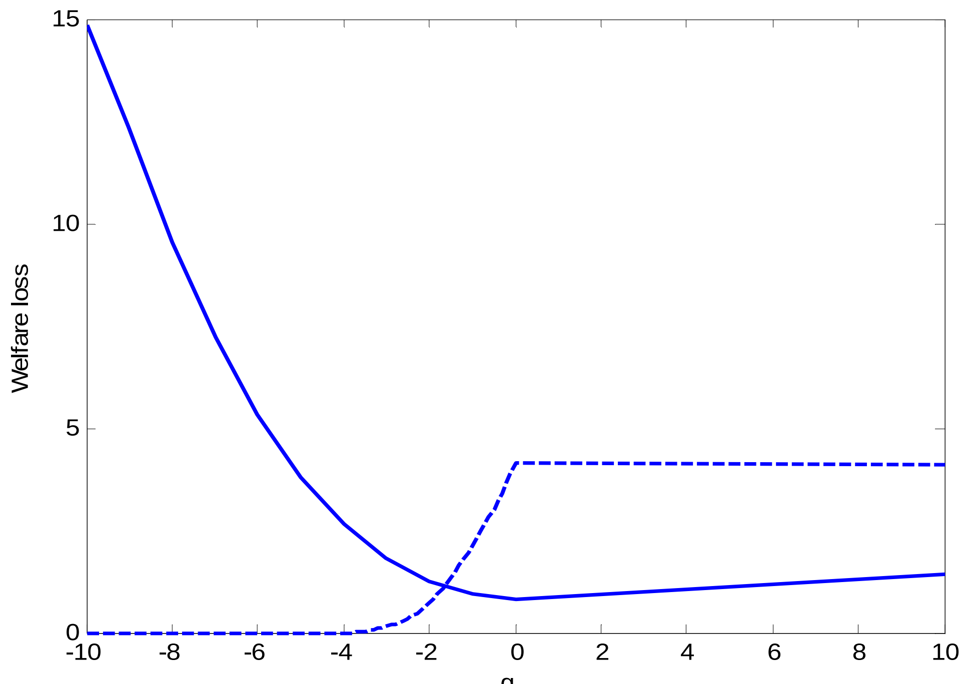

Figure 10. Welfare Loss with Output Shock
Dashed line-discretion; Solid line-rule

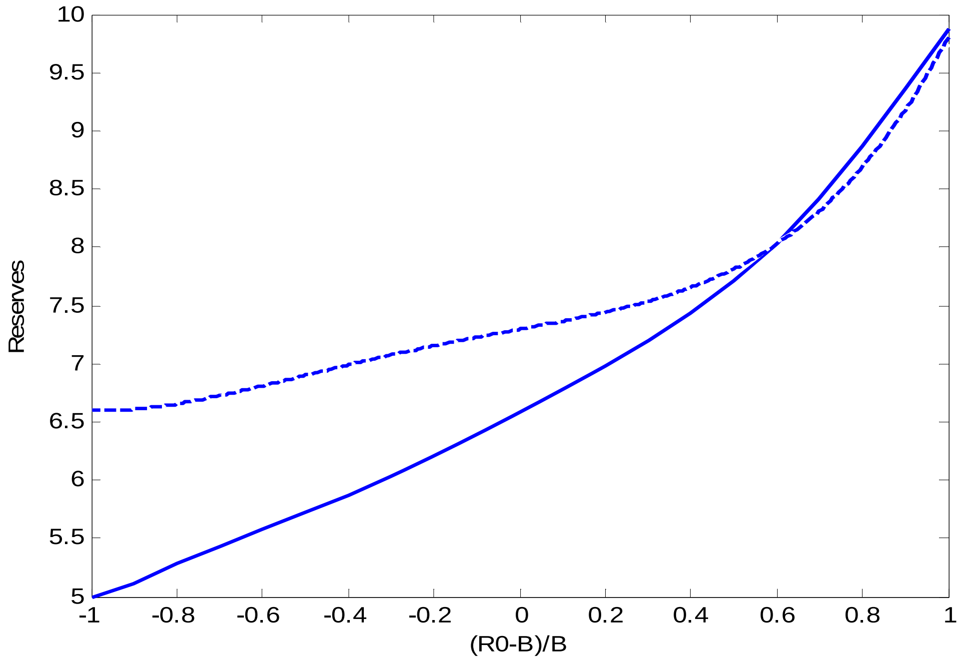

Figure 11. Reserves Dynamics under a Rule
Solid line – R_1; Dashed line-- $E(R_2)$

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